Assignment #1: Beauty in Mathematics

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The Spirograph

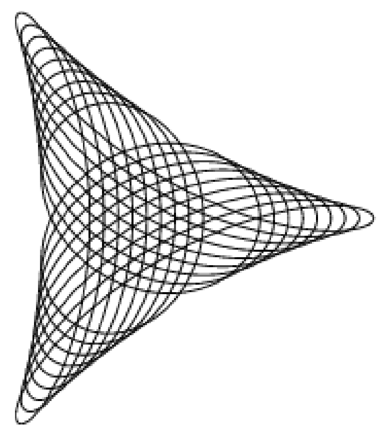
Remember the good old Spirograph?

The Spirograph

- The spirograph created pictures by tracing the path of a fixed point on a circle rotating inside another circle.
- The path of the fixed point can be traced by the parametric equations:

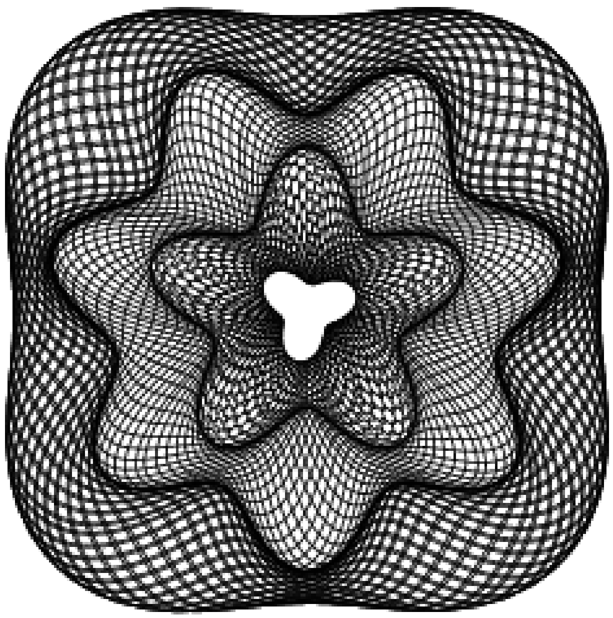
$$x = (R + r)\cos(\theta) - (r + \rho)\cos(\theta(R + r)/r)$$
$$y = (R + r)\sin(\theta) - (r + \rho)\sin(\theta(R + r)/r),$$

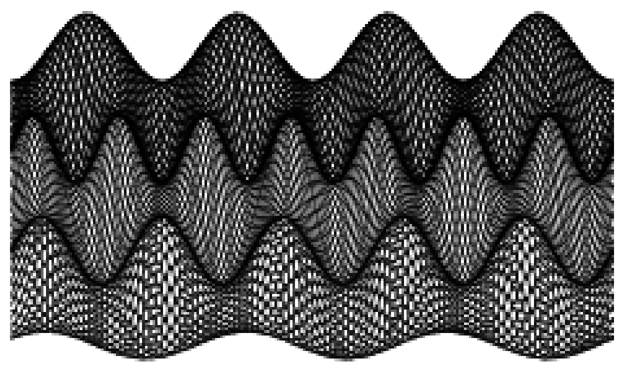
where R is the radius of the large circle and r is the radius of the small circle and ρ is the offset of the edge of the circle.



Guilloché Pattern

- Of course, spirograph was more than just a toy. Such drawings are used for several applications - most notably, banknotes and passports.
- The diagram shown below is called a Guilloché Pattern.



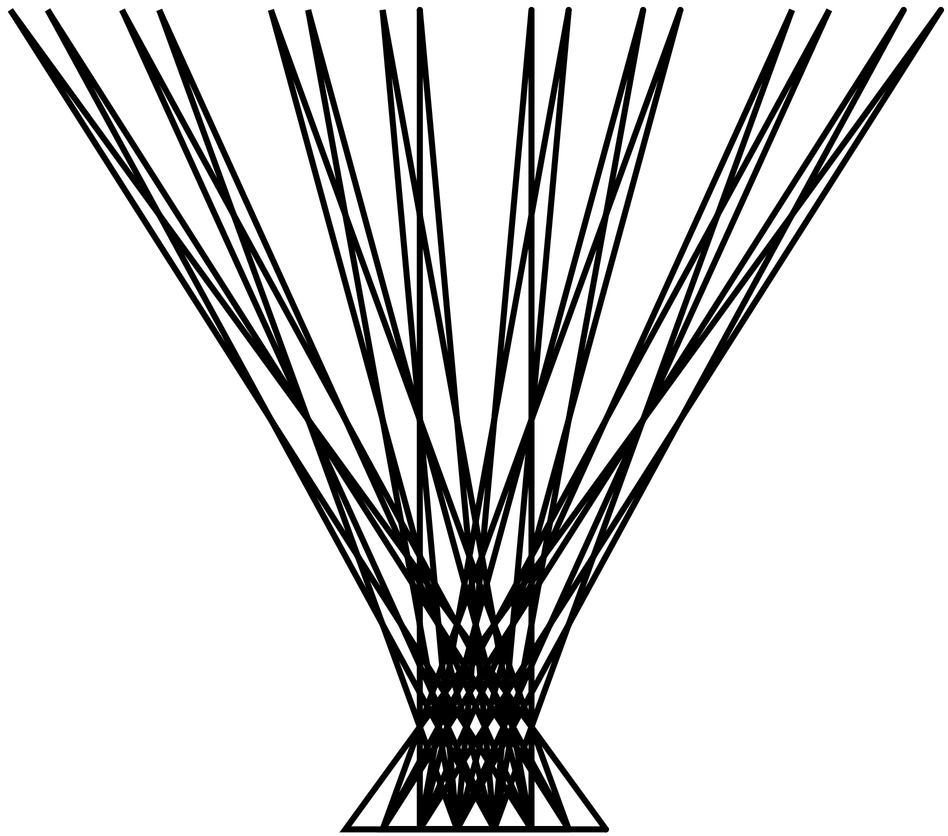


This is not, however, what I wish to discuss this morning.

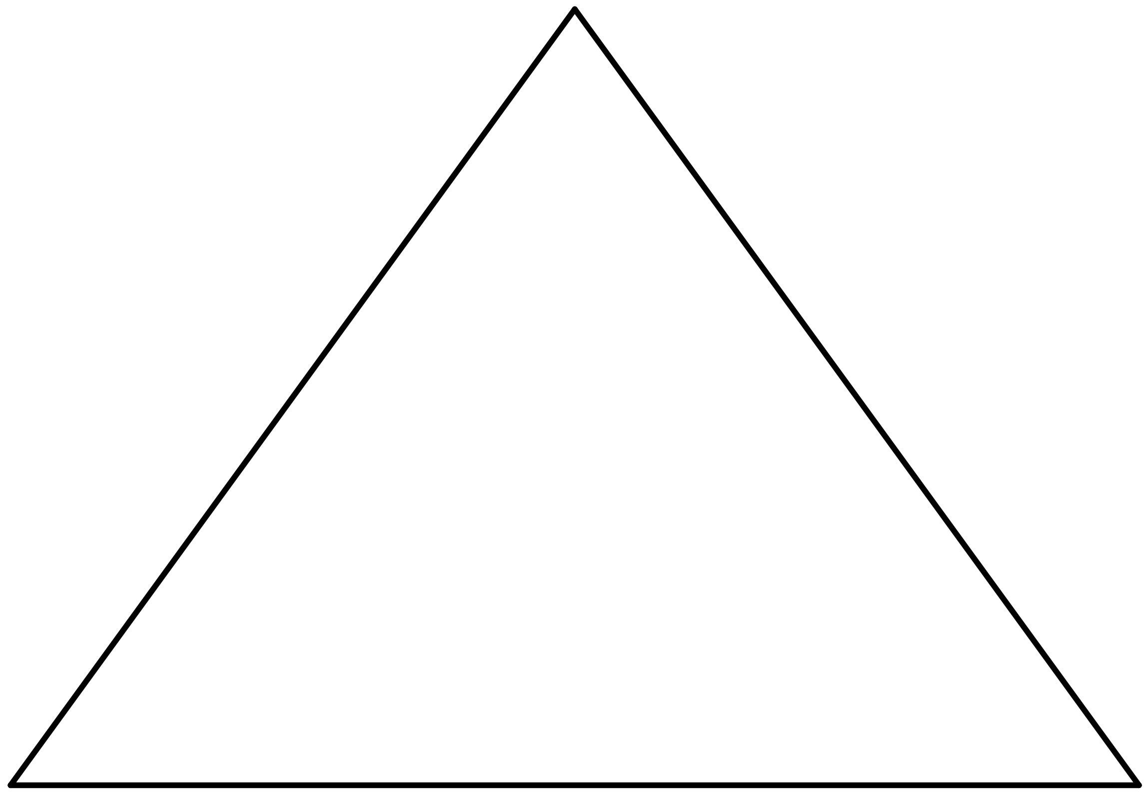
Clearly, there is something esthetically pleasing about these drawings. Something about the geometry and the perfect curves pleases our senses. Something similar is true for the human figure. The symmetry and perfect proportions of a beautiful person has much esthetic value. (Why is Barbie a North American Icon, then?)

Beauty in Symmetry

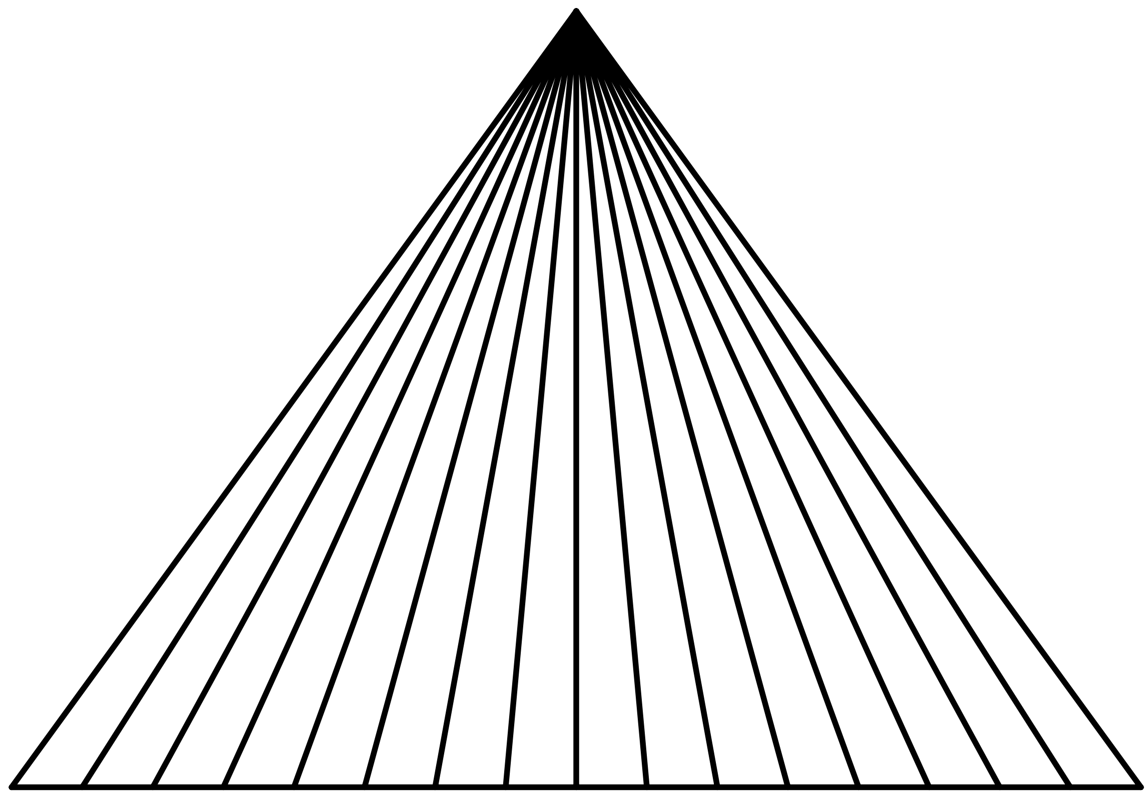
I propose that the following diagram has some beauty:



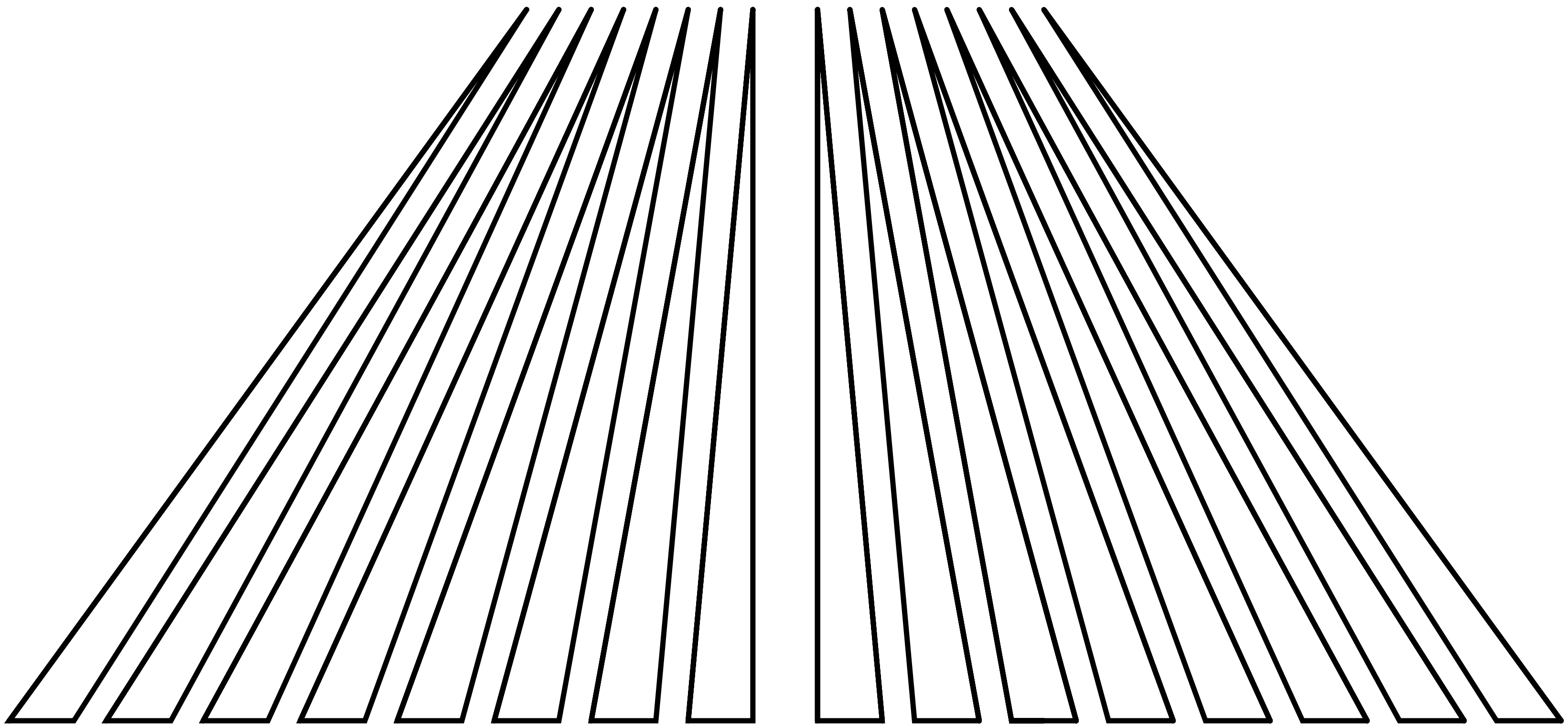
Beauty in Symmetry



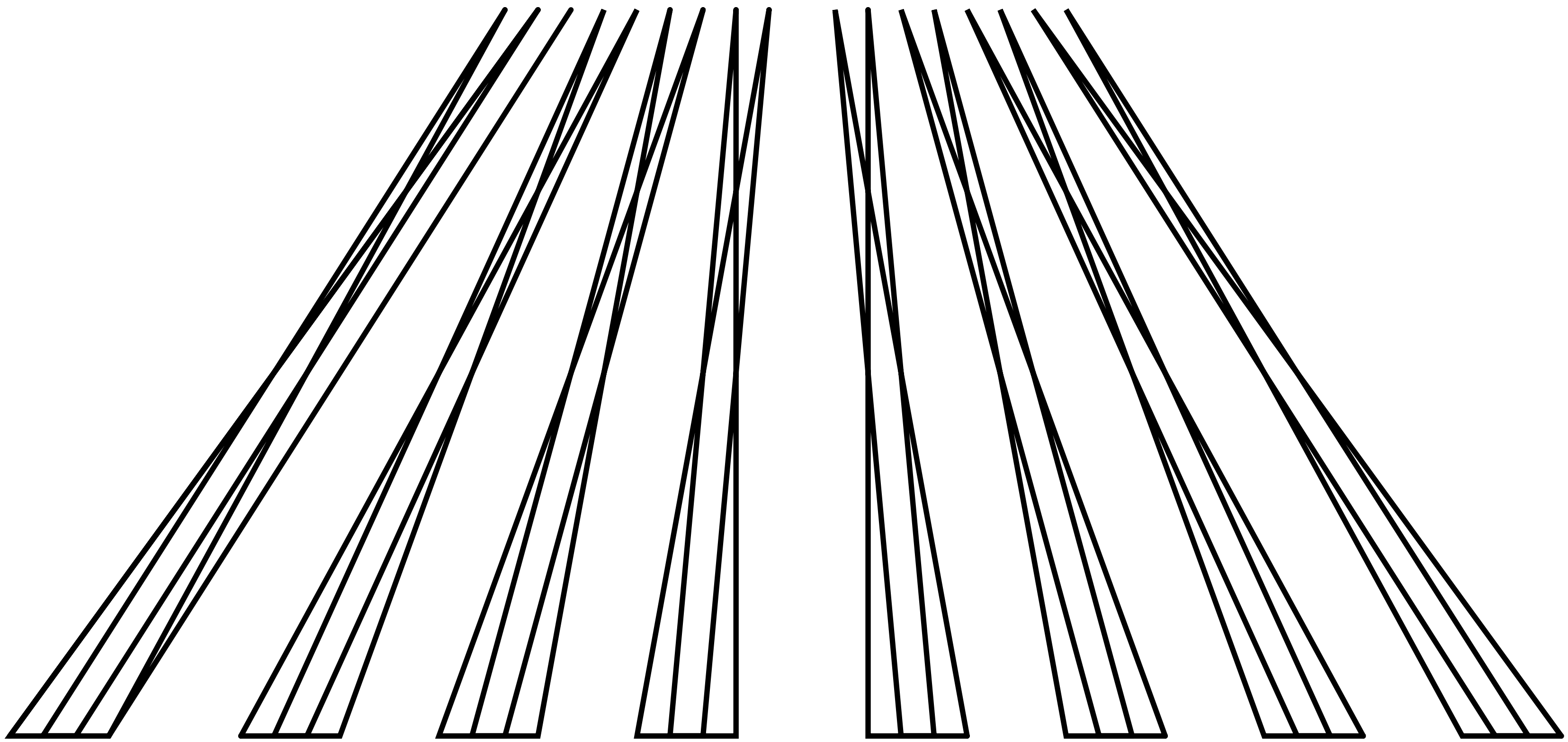
Beauty in Symmetry



Beauty in Symmetry



Beauty in Symmetry



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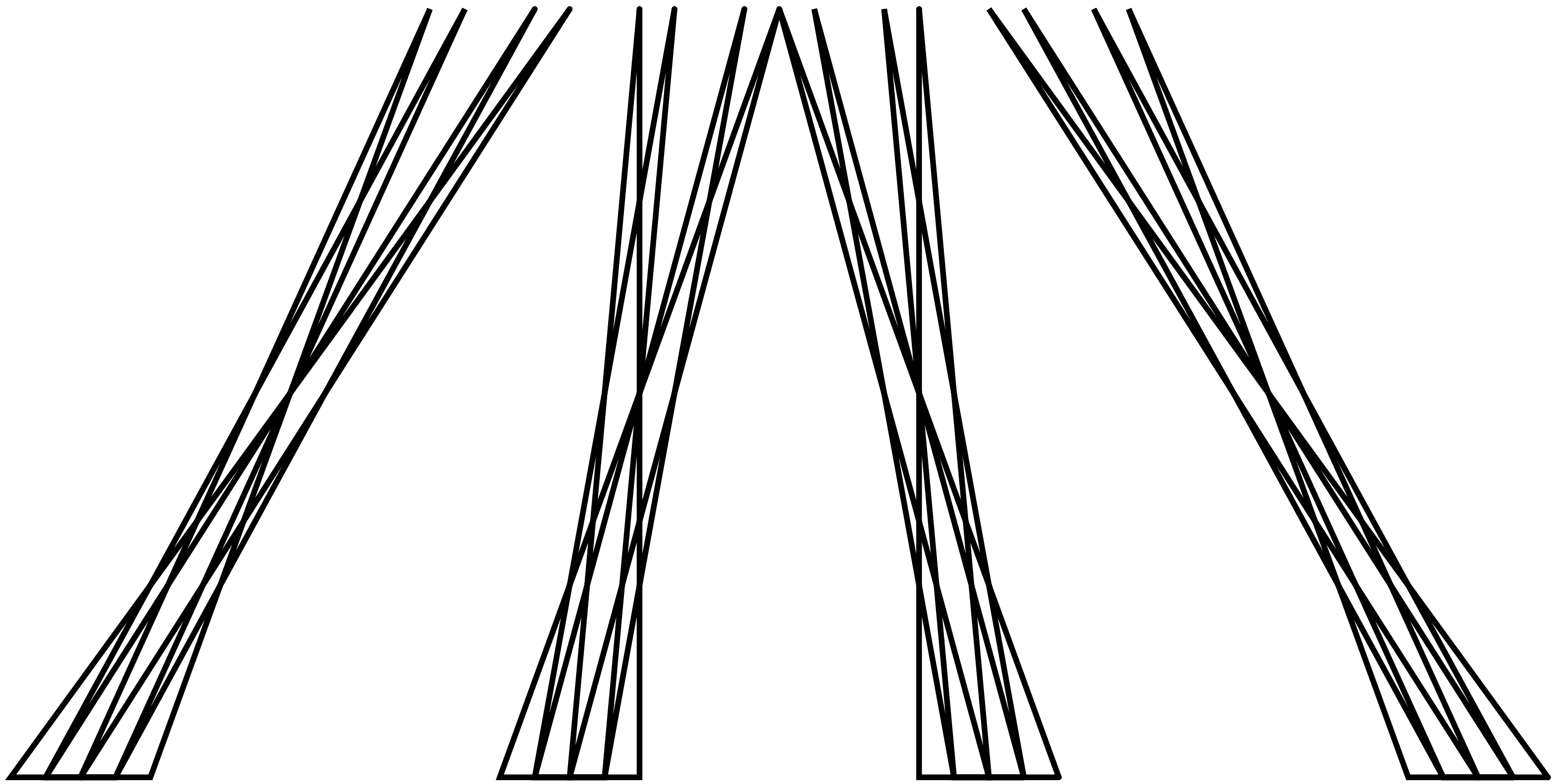

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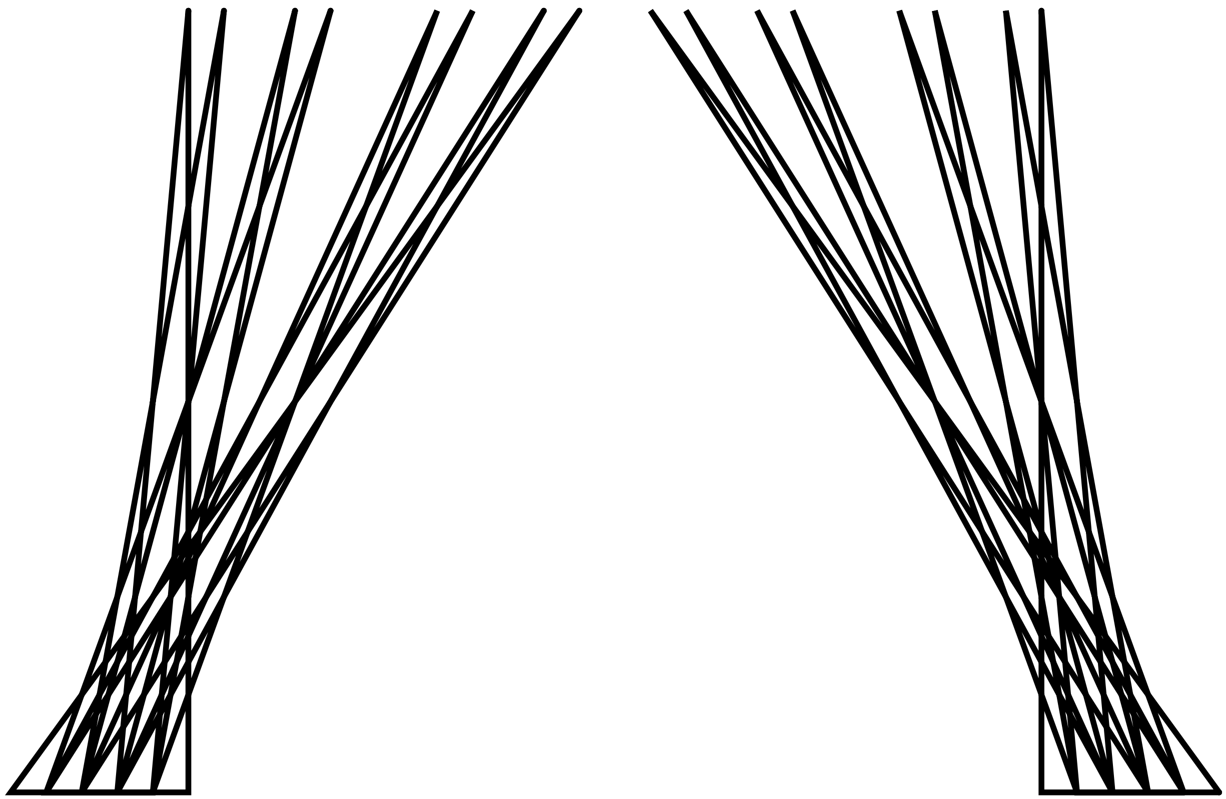

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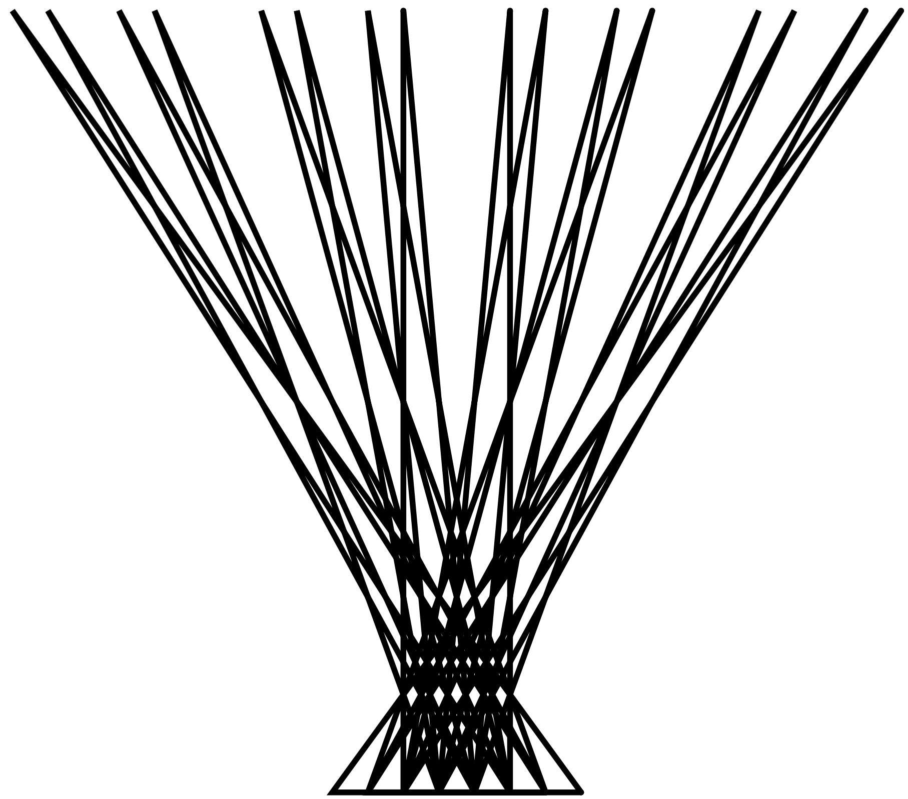
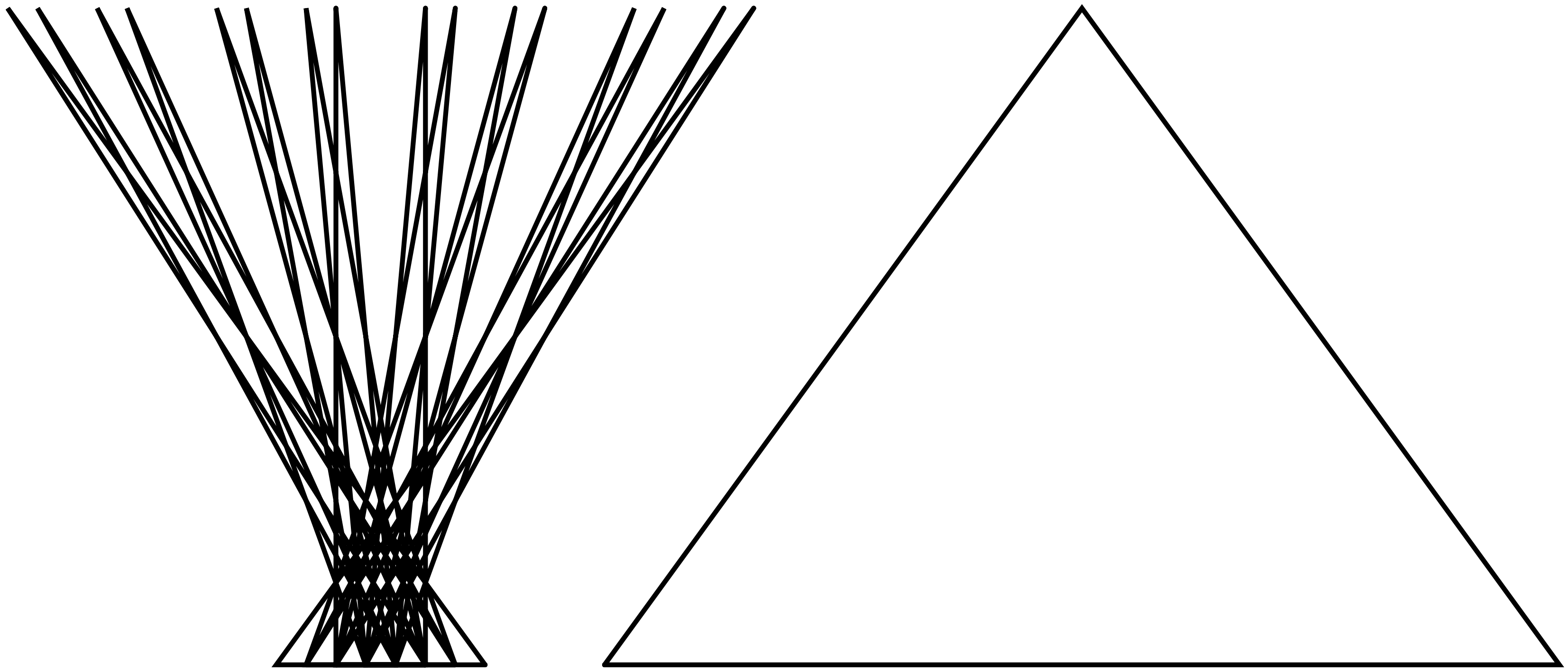


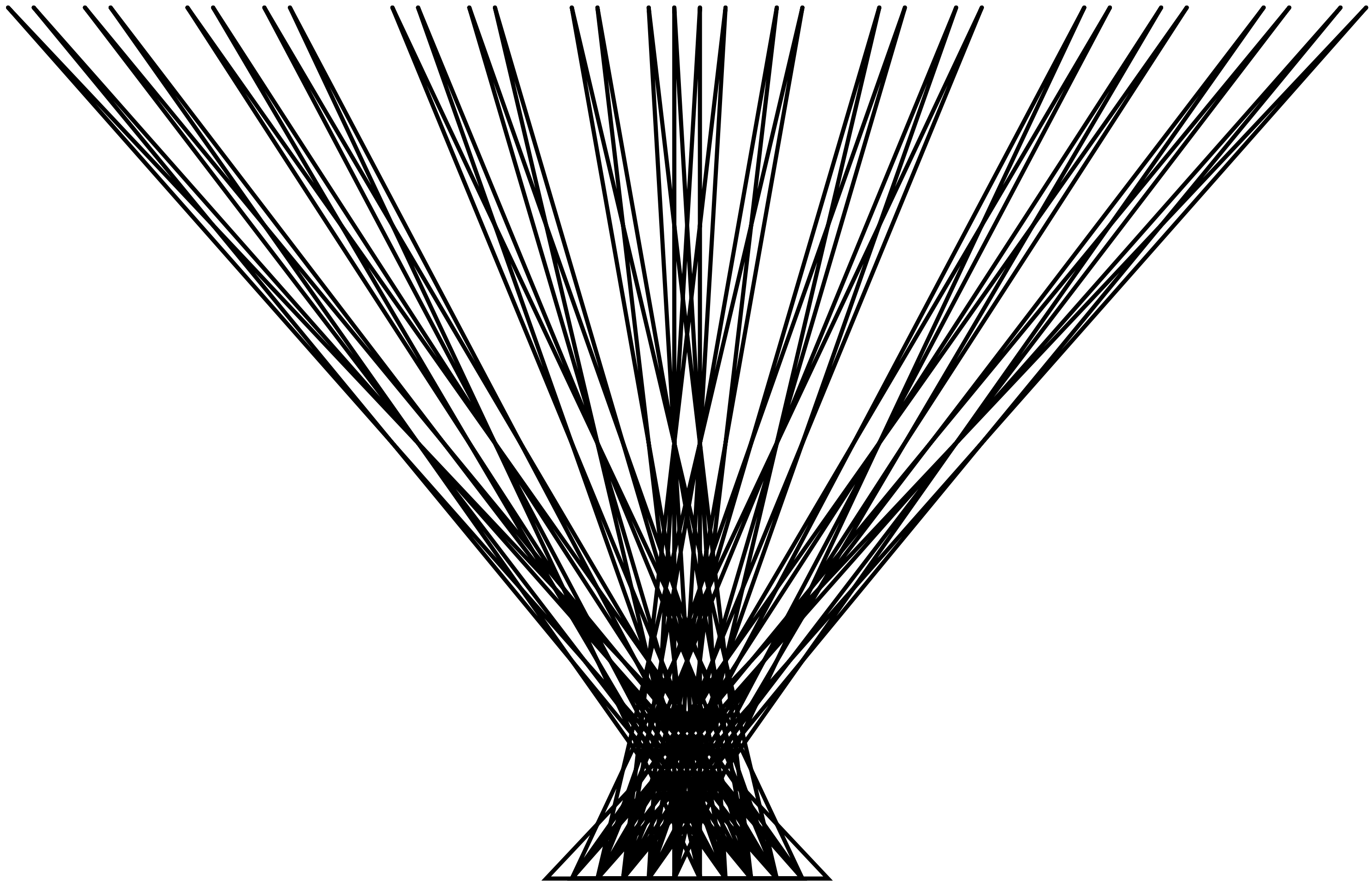
Beauty in Symmetry



The Kakeya Problem

- Given an n-dimensional pyramid, the problem of taking a pyramid, subdivided into n^k subsections, and rearranging these sections in such a way that it covers the least area/ volume etc. is known as the Kakeya problem.
- The solution to the 2D problem with 2^4 pieces is exactly the illustration above.
- Proofs of the higher dimensional cases are unknown; however, it is believed that (much like the 2D case) that it takes a large number of pieces.
- The fact that the proof leads to an illustration that is so easy to look at is rather beautiful in itself.

The Kakeya Problem - 32 subtriangles



Not Beautiful