MATHEMATICS 152 98-2 Solutions for Assignment 7

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2. If $u = e^x$ and $du = e^x dx$ then $\int e^{x+e^x}\,dx = \int e^x e^{e^x}\,dx = \int e^u\,du = e^u + C = e^{e^x} + C.$

10. If $u = \sqrt{1+\ln x}$ so that $\ln x = u^2 - 1$ and $\dfrac{dx}{x} = 2u\,du$ then

$$\int \frac{\sqrt{1+\ln x}}{x\ln x}\,dx = \int \frac{2u^2}{u^2-1}\,du = \int\left(2 + \frac{1}{u-1} - \frac{1}{u+1}\right)du = 2u + \ln|u-1| - \ln|u+1| + C =$$

$$= 2\sqrt{1+\ln x} + \ln\frac{\left|\sqrt{1+\ln x}-1\right|}{\sqrt{1+\ln x}+1} + C.$$

(Absolute value signs are not needed around $\sqrt{1+\ln x}+1$ since it cannot be smaller than 1.)

12. If $u = \tan x$ so that $du = \sec^2 x\,dx$, $u = 0$ when $x = 0$, and $u = 1$ when $x = \pi/4$, then $\int_0^{\pi/4} \tan^3 x \sec^4 x\,dx = \int_0^{\pi/4} \tan^3 x\,(1 + \tan^2 x)\sec^2 x\,dx = \int_0^1 (u^3 + u^5)\,du =$

$$= \left(\frac{1}{4}u^4 + \frac{1}{6}u^6\right)\Big]_0^1 = \frac{5}{12}.$$

16. Using partial fractions, $\displaystyle\int \frac{x}{x^2+3x+2}\,dx = \int \frac{x}{(x+1)(x+2)}\,dx = \int\left(\frac{-1}{x+1} + \frac{2}{x+2}\right)dx =$

$$= -\ln|x+1| + 2\ln|x+2| + C = \ln\frac{(x+2)^2}{|x+1|} + C.$$

28. $\displaystyle\int \tan^2(4x)\,dx = \int[\sec^2(4x) - 1]\,dx = \frac{1}{4}\tan(4x) - x + C.$

60. $\displaystyle\int \frac{dx}{x(x^4+1)} = \int \frac{(x^4+1)-x^4}{x(x^4+1)}\,dx = \int\left(\frac{1}{x} - \frac{x^3}{x^4+1}\right)dx = \ln|x| - \frac{1}{4}\ln(x^4+1) + C.$

This can also be done by a painful application of the method of partial fractions; the factor $(x^4 + 1)$ in the denominator can be written as $(x^2 + \sqrt{2}x + 1)(x^2 - \sqrt{2}x + 1)$ and then the algebra becomes even more unpleasant.

68. Since $1 + x - x^2 = \dfrac{5}{4} - \left(\dfrac{2x-1}{2}\right)^2$, $\sqrt{1+x-x^2} = \dfrac{\sqrt{5}}{2}\sqrt{1-\left(\dfrac{2x-1}{\sqrt{5}}\right)^2}$.

Putting $\dfrac{2x-1}{\sqrt{5}} = \sin t$ so that $\cos t = \dfrac{2\sqrt{1+x-x^2}}{\sqrt{5}}$ and $dx = \dfrac{\sqrt{5}}{2}\cos t\,dt$,

$$\int \sqrt{1+x-x^2}\,dx = \frac{5}{4}\int \cos^2 t\,dt = \frac{5}{8}\left(t + \sin t\cos t\right) + C = \frac{5}{8}\sin^{-1}\frac{2x-1}{\sqrt{5}} + \frac{(2x-1)\sqrt{1+x-x^2}}{4} + C.$$

80. If $u=\cos^2 x$ so that $du=-2\cos x\,\sin x\,dx=-\sin(2x)\,dx$, then
$$\int\frac{\sin(2x)}{\sqrt{9-\cos^4 x}}\,dx=-\int\frac{du}{\sqrt{9-u^2}}=-\sin^{-1}\frac{u}{3}+C=-\sin^{-1}\frac{\cos^2 x}{3}+C.$$

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2. If $u=x/2$ so that $dx=2\,du$, then by Formula 72 we have
$$\int\csc^3(x/2)\,dx=2\int\csc^3 u\,du=-\csc u\cot u+\ln\left|\csc u-\cot u\right|+C=$$
$$=-\csc(x/2)\cot(x/2)+\ln\left|\csc(x/2)-\cot(x/2)\right|+C.$$

8. If $u=x^2$ so that $du=2x\,dx$, then by Formula 90 we have
$$\int x^3\sin^{-1}(x^2)\,dx=\frac{1}{2}\int u\sin^{-1}u\,du=\frac{2u^2-1}{8}\sin^{-1}u+\frac{u\sqrt{1-u^2}}{8}+C=\frac{2x^4-1}{8}\sin^{-1}(x^2)+\frac{x^2\sqrt{1-x^4}}{8}+C.$$

12. By long division, $\dfrac{x^5}{x^2+\sqrt{2}}=x^3-\sqrt{2}\,x+\dfrac{2x}{x^2+\sqrt{2}}$.
If $u=x^2+\sqrt{2}$ so that $du=2x\,dx$, then
$$\int\frac{x^5}{x^2+\sqrt{2}}\,dx=\int\left(x^3-\sqrt{2}\,x+\frac{2x}{x^2+\sqrt{2}}\right)dx=\int(x^3-\sqrt{2}\,x)\,dx+\int\frac{du}{u}=$$
$$=\frac{1}{4}x^4-\frac{1}{\sqrt{2}}x^2+\ln|u|+C=\frac{1}{4}x^4-\frac{1}{\sqrt{2}}x^2+\ln\left(x^2+\sqrt{2}\right)+C,$$ by Formula 3.

14. If $u=2x$ so that $du=2\,dx$ then by repeatedly using Formula 73,
$$\int\sin^6(2x)\,dx=\frac{1}{2}\int\sin^6 u\,du=-\frac{1}{12}\sin^5 u\cos u+\frac{5}{12}\int\sin^4 u\,du=$$
$$=-\frac{1}{12}\sin^5 u\cos u-\frac{5}{48}\sin^3 u\cos u+\frac{5}{16}\int\sin^2 u\,du=$$
$$=-\frac{1}{12}\sin^5 u\cos u-\frac{5}{48}\sin^3 u\cos u-\frac{5}{32}\sin u\cos u+\frac{5}{32}\int 1\,du=$$
$$=-\frac{1}{12}\sin^5 u\cos u-\frac{5}{48}\sin^3 u\cos u-\frac{5}{32}\sin u\cos u+\frac{5}{32}u+C=$$
$$=-\frac{1}{12}\sin^5(2x)\cos(2x)-\frac{5}{48}\sin^3(2x)\cos(2x)-\frac{5}{32}\sin(2x)\cos(2x)+\frac{5}{16}x+C.$$

18. Write $x^2 - 4x = (x-2)^2 - 4$ and put $u = x - 2$, $a = 2$ so that $x^2 - 4x = u^2 - a^2$.
Then $du = dx$ and $\frac{x}{\sqrt{x^2-4x}} = \frac{u+2}{\sqrt{u^2-2^2}}$.
To integrate $\int \frac{x}{\sqrt{x^2-4x}}dx$ we will need to integrate $\int \frac{u}{\sqrt{u^2-2^2}}du$ and $\int \frac{2}{\sqrt{u^2-2^2}}du$.
To integrate the first of these two integrals, put $v = u^2 - 2^2$ so that $dv = 2u\,du$.
Then $\int \frac{u}{\sqrt{u^2-2^2}}du = \frac{1}{2}\int \frac{dv}{v^{1/2}} = v^{1/2} + C_1$.
To integrate the second, use Formula 43.
$\int \frac{2}{\sqrt{u^2-2^2}}du = 2\ln\left|u + \sqrt{u^2-2^2}\right| + C_2$. So $\int \frac{x}{\sqrt{x^2-4x}}dx = \sqrt{x^2-4x} + 2\ln\left|x - 2 + \sqrt{x^2-4x}\right| + C$.

26. If $u = \sin x$ so that $du = \cos x\,dx$ then $\int e^{\sin x}\sin(2x)\,dx = \int 2u e^u\,du$.
$\int e^{\sin x}\sin(2x)\,dx = \int 2ue^u\,du = 2(u-1)e^u + C = 2(\sin x - 1)e^{\sin x} + C$, by Formula 96.

28. Using the method of disks, if the region under the curve $y = \tan^2 x$ from $x = 0$ to $x = \frac{\pi}{4}$ is rotated about the x-axis, the volume of the resulting solid is
$V = \pi\int_0^{\pi/4}\tan^4 x\,dx = \pi\int_0^{\pi/4}\tan^2 x(\sec^2 x - 1)\,dx = \pi\int_0^{\pi/4}[\tan^2 x\sec^2 x - \tan^2 x]\,dx =$
$= \pi\int_0^{\pi/4}[\tan^2 x\sec^2 x - (\sec^2 x - 1)]\,dx = \pi\left(\frac{1}{3}\tan^3 x - \tan x + x\right)\Big]_0^{\pi/4} = \pi\left(\frac{\pi}{4} - \frac{2}{3}\right)$.

30. (a) $\frac{d}{du}\left(\frac{u}{8}(2u^2 - a^2)\sqrt{a^2-u^2} + \frac{a^4}{8}\sin^{-1}\frac{u}{a} + C\right) =$
$= \frac{1}{8}(2u^2-a^2)\sqrt{a^2-u^2} + \frac{u}{8}(4u)\sqrt{a^2-u^2} + \frac{u}{8}(2u^2-a^2)\frac{-u}{\sqrt{a^2-u^2}} + \frac{a^4}{8\sqrt{a^2-u^2}} =$
$= \frac{6u^2-a^2}{8}\sqrt{a^2-u^2} + \frac{-2u^4+a^2u^2+a^4}{8\sqrt{a^2-u^2}} = \frac{6u^2-a^2}{8}\sqrt{a^2-u^2} + \frac{(a^2-u^2)(a^2+2u^2)}{8\sqrt{a^2-u^2}} =$
$= \frac{6u^2-a^2}{8}\sqrt{a^2-u^2} + \frac{(a^2+2u^2)}{8}\sqrt{a^2-u^2} = u^2\sqrt{a^2-u^2}$.

(b) If $u = a\sin\theta$ then $\theta = \sin^{-1}\frac{u}{a}$, $du = a\cos\theta\,d\theta$, and $\sqrt{a^2-u^2} = a\cos\theta$.
$\int u^2\sqrt{a^2-u^2}\,du = \int(a\sin\theta)^2(a\cos\theta)(a\cos\theta\,d\theta) = a^4\int\sin^2\theta\cos^2\theta\,d\theta =$
$= \frac{a^4}{4}\int\sin^2(2\theta)\,d\theta = \frac{a^4}{8}\int[1-\cos(4\theta)]\,d\theta = \frac{a^4}{8}\left(\theta - \frac{1}{4}\sin(4\theta)\right) + C =$
$= \frac{a^4}{8}\left(\theta - \frac{1}{2}\sin(2\theta)\cos(2\theta)\right) + C = \frac{a^4}{8}[\theta - \sin\theta\cos\theta(1 - 2\sin^2\theta)] + C =$
$= \frac{a^4}{8}\left[\sin^{-1}\frac{u}{a} - \frac{u}{a}\frac{\sqrt{a^2-u^2}}{a}\left(1 - \frac{2u^2}{a^2}\right)\right] + C = \frac{a^4}{8}\sin^{-1}\frac{u}{a} - \frac{u}{8}(a^2 - 2u^2)\sqrt{a^2-u^2} + C$.

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8. Approximations to $\int_0^2 \frac{1}{\sqrt{1+x^3}}\,dx$; $n = 10$. (*Maple* estimates 1.402182105.)

$$T_{10} = \frac{1}{10}\left[\frac{1}{\sqrt{1+(0.0)^3}} + \frac{2}{\sqrt{1+(0.2)^3}} + \frac{2}{\sqrt{1+(0.4)^3}} + \frac{2}{\sqrt{1+(0.6)^3}} + \frac{2}{\sqrt{1+(0.8)^3}} + \frac{2}{\sqrt{1+(1.0)^3}} + \frac{2}{\sqrt{1+(1.2)^3}} + \right.$$

$$\left. + \frac{2}{\sqrt{1+(1.4)^3}} + \frac{2}{\sqrt{1+(1.6)^3}} + \frac{2}{\sqrt{1+(1.8)^3}} + \frac{1}{\sqrt{1+(2.0)^3}}\right] \approx 1.401435275.$$

$$M_{10} = \frac{1}{5}\left[\frac{1}{\sqrt{1+(0.1)^3}} + \frac{1}{\sqrt{1+(0.3)^3}} + \frac{1}{\sqrt{1+(0.5)^3}} + \frac{1}{\sqrt{1+(0.7)^3}} + \frac{1}{\sqrt{1+(0.9)^3}} + \frac{1}{\sqrt{1+(1.1)^3}} + \frac{1}{\sqrt{1+(1.3)^3}} + \right.$$

$$\left. + \frac{1}{\sqrt{1+(1.5)^3}} + \frac{1}{\sqrt{1+(1.7)^3}} + \frac{1}{\sqrt{1+(1.9)^3}}\right] \approx 1.402557804.$$

$$S_{10} = \frac{1}{15}\left[\frac{1}{\sqrt{1+(0.0)^3}} + \frac{4}{\sqrt{1+(0.2)^3}} + \frac{2}{\sqrt{1+(0.4)^3}} + \frac{4}{\sqrt{1+(0.6)^3}} + \frac{2}{\sqrt{1+(0.8)^3}} + \frac{4}{\sqrt{1+(1.0)^3}} + \frac{2}{\sqrt{1+(1.2)^3}} + \right.$$

$$\left. + \frac{4}{\sqrt{1+(1.4)^3}} + \frac{2}{\sqrt{1+(1.6)^3}} + \frac{4}{\sqrt{1+(1.8)^3}} + \frac{1}{\sqrt{1+(2.0)^3}}\right] \approx 1.402206274.$$

12. Approximations to $\int_0^1 \ln(1+e^x)\,dx$; $n = 8$. (*Maple* estimates 0.9838190370.)

$$T_8 = \frac{1}{16}\left[\ln(1+e^{0.000}) + 2\cdot\left\{\ln(1+e^{0.125}) + \ln(1+e^{0.250}) + \ln(1+e^{0.375}) + \ln(1+e^{0.500}) + \right.\right.$$

$$\left.\left. + \ln(1+e^{0.625}) + \ln(1+e^{0.750}) + \ln(1+e^{0.875})\right\} + \ln(1+e^{1.000})\right] \approx 0.984119925.$$

$\approx 0.984119925.$

$$M_8 = \frac{1}{8}\left[\ln(1+e^{0.0625}) + \ln(1+e^{0.1875}) + \ln(1+e^{0.3125}) + \ln(1+e^{0.4375}) + \ln(1+e^{0.5625}) + \right.$$

$$\left. + \ln(1+e^{0.6875}) + \ln(1+e^{0.8125}) + \ln(1+e^{0.9375})\right] \approx 0.983668581.$$

$$S_8 = \frac{1}{24}\left[\ln(1+e^{0.000}) + 4\ln(1+e^{0.125}) + 2\ln(1+e^{0.250}) + 4\ln(1+e^{0.375}) + 2\ln(1+e^{0.500}) + \right.$$

$$\left. + 4\ln(1+e^{0.625}) + 2\ln(1+e^{0.750}) + 4\ln(1+e^{0.875}) + \ln(1+e^{1.000})\right] \approx 0.983818913.$$

14. Approximations to $\int_0^4 \sqrt{x}\sin x\,dx$; $n = 8$. (*Maple* estimates 1.768748704.)

$$T_8 = \frac{1}{4}\left[\sqrt{0.0}\sin 0.0 + 2\cdot\left\{\sqrt{0.5}\sin 0.5 + \sqrt{1.0}\sin 1.0 + \sqrt{1.5}\sin 1.5 + \sqrt{2.0}\sin 2.0 + \right.\right.$$

$$\left.\left. + \sqrt{2.5}\sin 2.5 + \sqrt{3.0}\sin 3.0 + \sqrt{3.5}\sin 3.5\right\} + \sqrt{4.0}\sin 4.0\right] \approx 1.732865202.$$

$$M_8 = \frac{1}{2}\left[\sqrt{0.25}\sin 0.25 + \sqrt{0.75}\sin 0.75 + \sqrt{1.25}\sin 1.25 + \sqrt{1.75}\sin 1.75 + \sqrt{2.25}\sin 2.25 + \right.$$

$$\left. + \sqrt{2.75}\sin 2.75 + \sqrt{3.25}\sin 3.25 + \sqrt{3.75}\sin 3.75\right] \approx 1.787427119.$$

$$S_8 = \frac{1}{6}\left[\sqrt{0.0}\sin 0.0 + 4\sqrt{0.5}\sin 0.5 + 2\sqrt{1.0}\sin 1.0 + 4\sqrt{1.5}\sin 1.5 + 2\sqrt{2.0}\sin 2.0 + \right.$$

$$\left. + 4\sqrt{2.5}\sin 2.5 + 2\sqrt{3.0}\sin 3.0 + 4\sqrt{3.5}\sin 3.5 + \sqrt{4.0}\sin 4.0\right] \approx 1.772141513.$$

16. Approximations to $\int_2^4 \frac{e^x}{x}\,dx$; $n = 10$. (*Maple* estimates 14.67664011.)

$T_{10} = \frac{1}{10}\left[\frac{e^{2.0}}{2.0} + 2\left\{\frac{e^{2.2}}{2.2} + \frac{e^{2.4}}{2.4} + \frac{e^{2.6}}{2.6} + \frac{e^{2.8}}{2.8} + \frac{e^{3.0}}{3.0} + \frac{e^{3.2}}{3.2} + \frac{e^{3.4}}{3.4} + \frac{e^{3.6}}{3.6} + \frac{e^{3.8}}{3.8}\right\} + \frac{e^{4.0}}{4.0}\right] \approx$
$\approx 14.70459236.$

$M_{10} = \frac{1}{5}\left[\frac{e^{2.1}}{2.1} + \frac{e^{2.3}}{2.3} + \frac{e^{2.5}}{2.5} + \frac{e^{2.7}}{2.7} + \frac{e^{2.9}}{2.9} + \frac{e^{3.1}}{3.1} + \frac{e^{3.3}}{3.3} + \frac{e^{3.5}}{3.5} + \frac{e^{3.7}}{3.7} + \frac{e^{3.9}}{3.9}\right] \approx$
$\approx 14.66266926.$

$S_{10} = \frac{1}{15}\left[\frac{e^{2.0}}{2.0} + 4\frac{e^{2.2}}{2.2} + 2\frac{e^{2.4}}{2.4} + 4\frac{e^{2.6}}{2.6} + 2\frac{e^{2.8}}{2.8} + 4\frac{e^{3.0}}{3.0} + 2\frac{e^{3.2}}{3.2} + 4\frac{e^{3.4}}{3.4} + 2\frac{e^{3.6}}{3.6} + 4\frac{e^{3.8}}{3.8} + \frac{e^{4.0}}{4.0}\right] \approx$
$\approx 14.67669609.$

18. Approximations to $\int_0^1 \cos(x^2)\,dx$; $n = 4$ or 8. (*Maple* estimates 0.9045242379.)

$T_4 = \frac{1}{8}\left[\cos(0.00)^2 + 2\cdot\{\cos(0.25)^2 + \cos(0.50)^2 + \cos(0.75)^2\} + \cos(1.00)^2\right] \approx$
$\approx 0.895758896.$

$T_8 = \frac{1}{16}\left[(\cos(0.000)^2 + 2\cdot\{\cos(0.125)^2 + \cos(0.250)^2 + \cos(0.375)^2 + \cos(0.500)^2 +\right.$
$\left. + \cos(0.625)^2 + \cos(0.750)^2 + \cos(0.875)^2\} + \cos(1.000)^2\right] \approx 0.902332843.$

$M_4 = \frac{1}{4}\left[\cos(0.125)^2 + \cos(0.375)^2 + \cos(0.625)^2 + \cos(0.875)^2\right] \approx 0.908906791.$

$M_8 = \frac{1}{8}\left[\cos(0.0625)^2 + \cos(0.1875)^2 + \cos(0.3125)^2 + \cos(0.4375)^2 +\right.$
$\left. + \cos(0.5625)^2 + \cos(0.6875)^2 + \cos(0.8125)^2 + \cos(0.9375)^2\right] \approx 0.905619957.$

If $f(x) = \cos(x^2)$, then $f'(x) = -2x\sin(x^2)$ and $f''(x) = -2\sin(x^2) - 4x^2\cos(x^2)$.
On $[0, 1]$ we have the bound $|f''(x)| \le 6 = M_2$. You can see that $|f''(x)| \le 6$ because $|x^2| \le 1$, $|\sin(x^2)| \le 1$, and $|\cos(x^2)| \le 1$.

$|E_T| \le \frac{M_2(b-a)^3}{12n^2} = \frac{6\cdot 1^3}{12n^2} = \frac{1}{2n^2}$. When $n = 4$, $|E_T| \le \frac{1}{32} = 0.03125$. When $n = 8$, $|E_T| \le \frac{1}{128} = 0.0078125$. Using the *Maple* estimate for the integral, when $n = 4$, $E_T \approx 0.008765341$, and when $n = 8$, $E_T \approx 0.002191394$.

$|E_M| \le \frac{M_2(b-a)^3}{24n^2} = \frac{6\cdot 1^3}{24n^2} = \frac{1}{4n^2}$. When $n = 4$, $|E_M| = \frac{1}{64} = 0.015625$. When $n = 8$, $|E_M| = \frac{1}{256} = 0.00390625$. Using the *Maple* estimate for the integral, when $n = 4$, $E_M \approx -0.004382553$, and when $n = 8$, $E_M \approx -0.001095719$.

Notice that our error bounds for E_T and E_M are nearly 4 times as large as the actual errors. Most of the time $|f''(x)|$ is nowhere near as large as 6. For one thing, we are not making use of the fact that there is no way that $|\sin(x^2)|$ and $|\cos(x^2)|$ can be 1 simultaneously, the pessimistic assumption made in our estimate for M_2. The algebraic sign of the E_M is opposite to the sign of E_T, as it should be, and the size of E_T is almost exactly twice that for E_M. This should make us expect Simpson's Rule to work very well for this function, so let's try it although the exercise does not require this.

$S_4 = \frac{1}{12}\left[\cos(0.00)^2 + 4\cdot\cos(0.25)^2 + 2\cdot\cos(0.50)^2 + 4\cdot\cos(0.75)^2 + \cos(1.00)^2\right] \approx$
$\approx 0.904501266.$

$S_8 = \frac{1}{24}\left[\cos(0.000)^2 + 4\cdot\cos(0.125)^2 + 2\cdot\cos(0.250)^2 + 4\cdot\cos(0.375)^2 +\right.$
$\left.+ 2\cdot\cos(0.500)^2 + 4\cdot\cos(0.625)^2 + 2\cdot\cos(0.750)^2 + 4\cdot\cos(0.875)^2 + \cos(1.000)^2\right] \approx$
$\approx 0.904524159.$

$f^{(3)}(x) = -12x\cos(x^2) + 8x^3\sin(x^2)$, $f^{(4)}(x) = -12\cos(x^2) + 48x^2\sin(x^2) + 16x^4\cos(x^2)$, and on $[0, 1]$ we have the bound $\left|f^{(4)}(x)\right| \le 60 = M_4$. Since $0 \le x^4 \le 1$, you can see that $-12 \le -12 + 16x^4 \le 4$, so $\left|(-12+16x^4)\cos(x^2)\right| \le 12$ and $\left|48x^2\sin(x^2)\right| \le 48$.

$\left|E_S\right| \le \frac{M_4(b-a)^5}{180n^4} = \frac{60\cdot 1^5}{180n^4} = \frac{1}{3n^4}$. When $n = 4$, $\left|E_S\right| \le \frac{1}{768} \approx 0.001302083$. When $n = 8$, $\left|E_S\right| \le \frac{1}{12288} \approx 0.0000813802083$. Using the *Maple* estimate for the integral, when $n = 4$, $E_S \approx -0.000022972$ and when $n = 8$, $E_S \approx -0.000000079$. Simpson's Rule is working much better than advertised!

22. If $f(x) = e^{x^2}$ then $f'(x) = 2xe^{x^2}$, $f''(x) = (4x^2 + 2)e^{x^2}$, $f^{(3)}(x) = (8x^3 + 12x)e^{x^2}$, and $f^{(4)}(x) = (16x^4 + 48x^2 + 12)e^{x^2}$. So on the interval $0 \le x \le 1$, the largest value of $f^{(4)}(x)$ is clearly $f^{(4)}(1) = 76e \approx 206.589419$. Since $\left|E_S\right| \le \frac{M_4(b-a)^5}{180n^4} = \frac{76e\cdot 1^5}{180n^4} = \frac{19e}{45n^4}$, to insure that $\left|E_S\right| \le 0.00001$ we will want to have $n^4 > \frac{1900000\,e}{45} \approx 114771.8994$.

$n = 18$ doesn't quite do it, but $n = 20$ does.

(Odd values of n make no sense in Simpson's Rule, so we can't use $n = 19$.)

I calculated $S_{20} \approx 1.462653625$ in somewhat less than a minute using a TI–36 calculator. My Macintosh Quadra running *Maple*, set for 20 digits accuracy, in just under two seconds estimated $\int_0^1 e^{x^2}dx \approx 1.4626517459071816088$. So when $n = 20$, $E_S \approx -0.000001879$, more than 5 times as accurate as our guarantee.

If you look at the formula for $f^{(4)}(x)$ you can see that over most of the interval $0 \le x \le 1$, it is nowhere near as large as its maximum value. For example the minimum value is $f^{(4)}(0) = 12$. This is why our accuracy is so much better than the error bound indicates. Probably a much smaller value than $n = 20$ would have been adequate.