38. $\int x e^{x^2}\,dx = \frac{1}{2} e^{x^2} + C$. (Let $u = x^2$, $du = 2x\,dx$.)

40. $\int \frac{\tan^{-1} x}{1 + x^2}\,dx = \frac{1}{2}(\tan^{-1} x)^2 + C$. (Let $u = \tan^{-1} x$, $du = \frac{1}{1 + x^2}\,dx$.)

42. $\int e^x \sin(e^x)\,dx = -\cos(e^x) + C$. (Let $u = e^x$, $du = e^x dx$.)

46. $\int \frac{\sin x}{1 + \cos^2 x}\,dx = -\tan^{-1}(\cos x) + C$. (Let $u = \cos x$, $du = -\sin x\,dx$.)

70. Let $u = x^2 + a^2$, so $du = 2x\,dx$. When $x = -a$, $u = 2a^2$. When $x = a$, $u = 2a^2$.
$\int_{-a}^{a} x\sqrt{x^2 + a^2}\,dx = \frac{1}{2}\int_{2a^2}^{2a^2} u^{1/2}\,du = \frac{1}{3}\,u^{3/2}\Big]_{2a^2}^{2a^2} = 0.$
Of course you can see this from the symmetry also.

78. Let $u = x^2$, so $du = 2x\,dx$, and $\int_0^1 x\sqrt{1 - x^4}\,dx = \frac{1}{2}\int_0^1 \sqrt{1 - u^2}\,du$.
This can be interpreted as half the area in the first quadrant inside the unit circle, $\frac{\pi}{8}$.

80. The period from the beginning of the third week to the end of the fourth week is from the end of the second week to the beginning of the fourth week
$\int_2^4 5000\left(1 - \frac{100}{(t + 10)^2}\right) dt = 5000\left(t + \frac{100}{t + 10}\right)\Big]_2^4 = \frac{85000}{21}$.
About 4048 calculators are produced.

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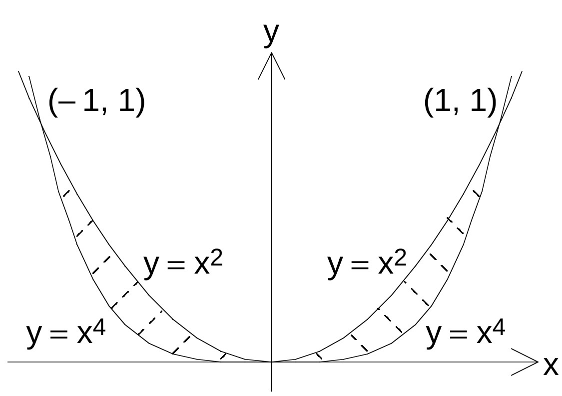
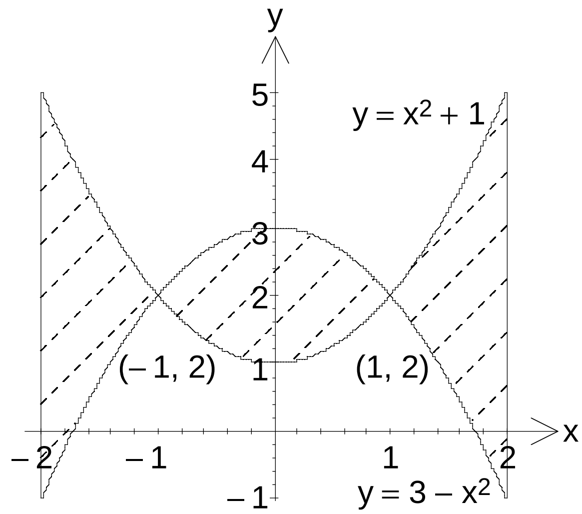
8. The curves $y = x^2$ and $y = x^4$ meet at $(-1, 1)$, $(0, 0)$, and $(1, 1)$ with $y = x^2$ above $y = x^4$ both when $-1 < x < 0$ and when $0 < x < 1$.
The area between the curves is
$\int_{-1}^{1} (x^2 - x^4)\,dx = \left(\frac{1}{3}x^3 - \frac{1}{5}x^5\right)\Big]_{-1}^{1} = \frac{4}{15}$.
See graph to the right.

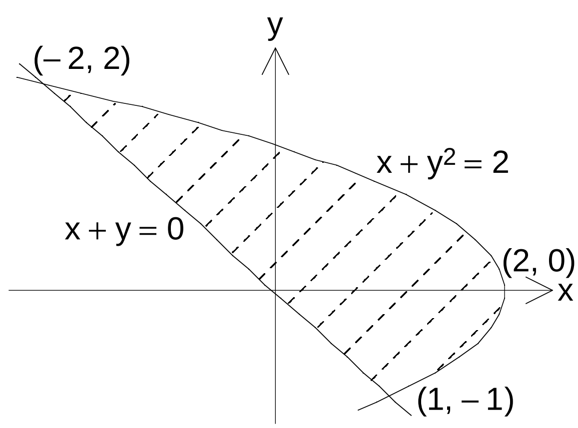


14. The two parabolas $y = x^2 + 1$ and $y = 3 - x^2$ meet at $(-1, 2)$ and $(1, 2)$. The curve $y = x^2 + 1$ is uppermost for $-2 \le x < -1$ and for $1 < x \le 2$, while the curve $y = 3 - x^2$ is uppermost for $-1 < x < 1$. The portion of the area between the curves and between the lines $x = -2$ and $x = -1$ is



$\int_{-2}^{-1}(2x^2 - 2)\,dx = \left(\frac{2}{3}x^3 - 2x\right)\Big]_{-2}^{-1} = \frac{8}{3}$.
The portion of the area between the curves between $x = -1$ and $x = 1$ is
$\int_{-1}^{1}(2 - 2x^2)\,dx = \left(2x - \frac{2}{3}x^3\right)\Big]_{-1}^{1} = \frac{8}{3}$.
The portion of the area between the curves between $x = 1$ and $x = 2$ is
$\int_{1}^{2}(2x^2 - 2)\,dx = \left(\frac{2}{3}x^3 - 2x\right)\Big]_{1}^{2} = \frac{8}{3}$. Thus the total area is 8.

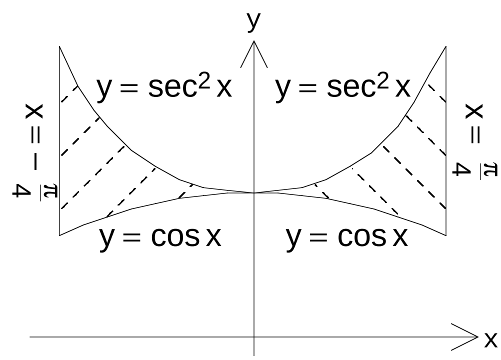
16. The parabola $x + y^2 = 2$ and the line $x + y = 0$ meet at $(-2, 2)$ and at $(1, -1)$ with the parabola above and to the right of the line between these two points.
The parabola vertex is at $(2, 0)$.
The area between the curves is



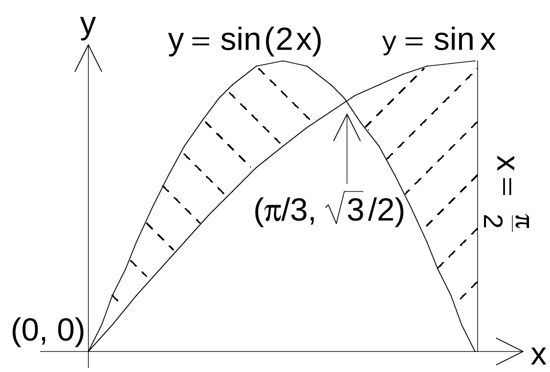
$\int_{-1}^{2}((2 - y^2) - (-y))\,dy =$
$= \int_{-1}^{2}(-y^2 + y + 2)\,dy =$
$= \left(-\frac{1}{3}y^3 + \frac{1}{2}y^2 + 2y\right)\Big]_{-1}^{2} = \frac{9}{2}$.

Alternatively, break the region into two pieces, $-2 \le x \le 1$ and $1 \le x \le 2$, using $y = -x$ and $y = \sqrt{2-x}$ as lower and upper boundary curves for $-2 \le x \le 1$, and using $y = -\sqrt{2-x}$ and $y = \sqrt{2-x}$ as the boundary curves for $1 \le x \le 2$.
The area is $\int_{-2}^{1} \left(\sqrt{2-x} - (-x)\right) dx + \int_{1}^{2} \left(\sqrt{2-x} - (-\sqrt{2-x})\right) dx =$
$= \int_{-2}^{1} ((2-x)^{1/2} + x)\, dx + \int_{1}^{2} 2(2-x)^{1/2} dx = \left(-\frac{2}{3}(2-x)^{3/2} + \frac{1}{2}x^2\right)\Big]_{-2}^{1} - \frac{4}{3}(2-x)^{3/2}\Big]_{1}^{2} = \frac{9}{2}$.
See graph on the preceding page.

20. The curve $y = \sec^2 x$ is above the curve $y = \cos x$ for both $-\frac{\pi}{4} \le x < 0$ and $0 < x \le \frac{\pi}{4}$.
The area is $\int_{-\pi/4}^{\pi/4} (\sec^2 x - \cos x)\, dx =$
$= (\tan x - \sin x)\Big]_{-\pi/4}^{\pi/4} = 2 - \sqrt{2}$.
See graph to the right.



22. In the region where $0 \le x \le \frac{\pi}{2}$,
the curves $y = \sin x$ and $y = \sin(2x)$ meet where $2 \sin x \cos x = \sin x$, or where either $\sin x = 0$ or $\cos x = \frac{1}{2}$,
at $(0, 0)$ and $(\pi/3, \sqrt{3}/2)$, with $y = \sin(2x)$ above $y = \sin x$ if $0 < x < \frac{\pi}{3}$ and with $y = \sin x$ above $y = \sin(2x)$ if $\frac{\pi}{3} < x \le \frac{\pi}{2}$.
The area is $\int_{0}^{\pi/2} |\sin(2x) - \sin x|\, dx =$
$= \int_{0}^{\pi/3} [\sin(2x) - \sin x]\, dx + \int_{\pi/3}^{\pi/2} [(\sin x - \sin(2x)]\, dx =$
$= \left(-\frac{1}{2}\cos(2x) + \cos x\right)\Big]_{0}^{\pi/3} + \left(-\cos x + \frac{1}{2}\cos(2x)\right)\Big]_{\pi/3}^{\pi/2} = \frac{1}{2}$.
See graph above and to the right.



30. The area of the region is

$$\int_0^{1/2}(2-1)\,dx+\int_{1/2}^1\left(\frac{1}{x}-1\right)dx=$$

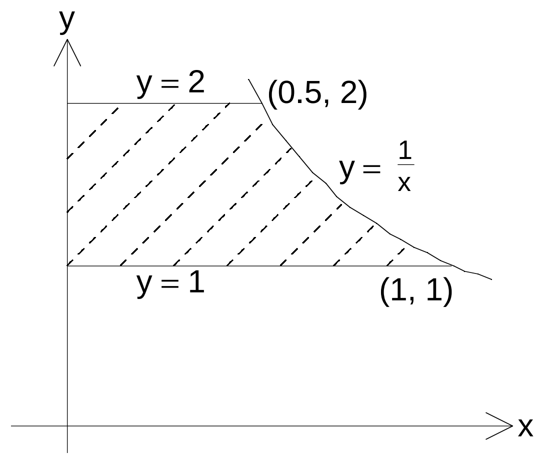
$$=\int_0^{1/2}1\,dx+\int_{1/2}^1\left(\frac{1}{x}-1\right)dx=$$

$$=x\Big]_0^{1/2}+(\ln x-x)\Big]_{1/2}^1=-\ln\frac{1}{2}=\ln 2.$$

Alternatively integrating with respect to y, the area of the region is

$$\int_1^2\left(\frac{1}{y}-0\right)dy=\ln y\Big]_1^2=\ln 2.$$

See graph to the right.



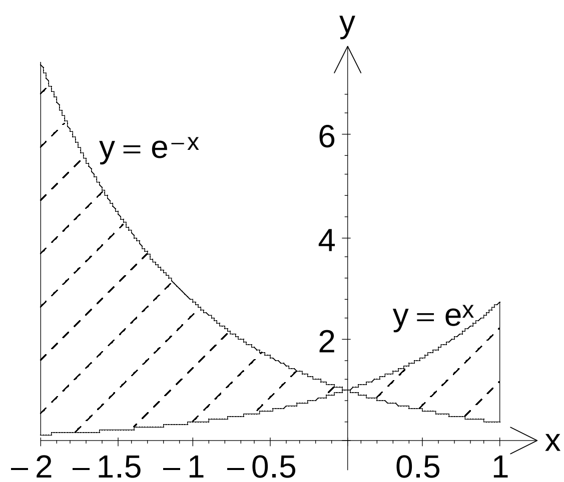
34. The two curves $y=e^x$ and $y=e^{-x}$ meet at $(0, 1)$. For $-2 \le x < 0$, the curve $y=e^{-x}$ is uppermost, while for $0 < x \le 1$, the curve $y=e^x$ is uppermost.
The area of the region between these two curves and between $x=-2$ and $x=1$ is

$$\int_{-2}^0[e^{-x}-e^x]dx+\int_0^1[e^x-e^{-x}]\,dx=$$

$$=[-e^{-x}-e^x]\Big]_{-2}^0+[e^x+e^{-x}]\Big]_0^1=$$

$$=e^2+e+\frac{1}{e}+\frac{1}{e^2}-4.$$

See graph to the right.



36. The line $x+6y=7$ meets the **lower** half of the parabola $x+1=2(y-2)^2$ at $(1, 1)$ and $(7, 0)$, with the line above and to the right of the parabola between those points.

The area is $\int_1^7\left(\frac{7-x}{6}-\left(2-\frac{\sqrt{x+1}}{\sqrt{2}}\right)\right)dx=$

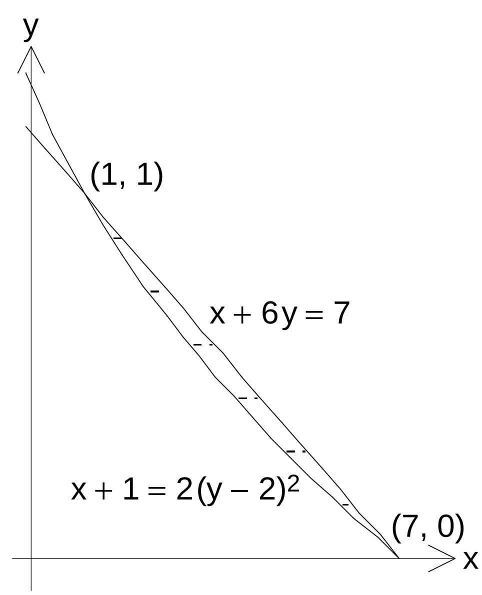
$$=\int_1^7\left(\frac{1}{\sqrt{2}}(x+1)^{1/2}-\frac{1}{6}x-\frac{5}{6}\right)dx=$$

$$=\left(\frac{\sqrt{2}}{3}(x+1)^{3/2}-\frac{1}{12}x^2-\frac{5}{6}x\right)\Bigg]_1^7=\frac{1}{3}.$$

Alternatively we can integrate with respect to y.
The area is $\int_0^1[(7-6y)-(2(y-2)^2-1)]\,dy=$

$$=\int_0^1(-2y^2+2y)\,dy=\left(-\frac{2}{3}y^3+y^2\right)\Big]_0^1=\frac{1}{3}.$$

See graph to the right.



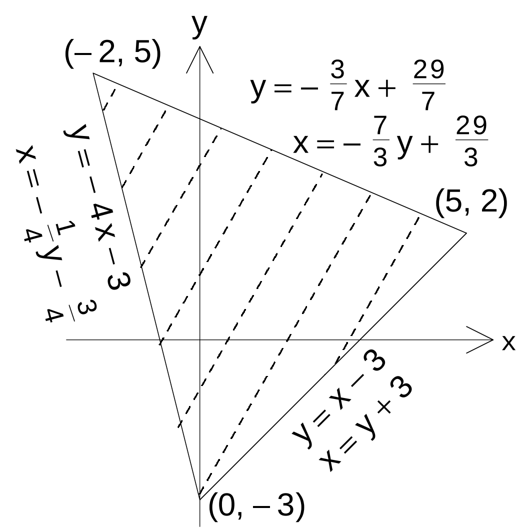
38. The line segment through the points $(-2, 5)$ and $(5, 2)$ has equations $y = -\frac{3}{7}x + \frac{29}{7}$ and $x = -\frac{7}{3}y + \frac{29}{3}$.
The line segment through the points $(-2, 5)$ and $(0, -3)$ has equations $y = -4x - 3$ and $x = -\frac{1}{4}y - \frac{3}{4}$.
The line segment through the points $(0, -3)$ and $(5, 2)$ has equations $y = x - 3$ and $x = y + 3$.
The area of the triangle is

$$\int_{-2}^{0}\left(\frac{25}{7}x + \frac{50}{7}\right)dx + \int_{0}^{5}\left(-\frac{10}{7}x + \frac{50}{7}\right)dx =$$
$$= \left(\frac{25}{14}x^2 + \frac{50}{7}x\right)\Big]_{-2}^{0} + \left(-\frac{5}{7}x^2 + \frac{50}{7}x\right)\Big]_{0}^{5} = 25.$$

Alternatively the area is

$$\int_{-3}^{2}\left(\frac{5}{4}y + \frac{15}{4}\right)dy + \int_{2}^{5}\left(-\frac{25}{12}y + \frac{125}{12}\right)dy =$$
$$= \left(\frac{5}{8}y^2 + \frac{15}{4}y\right)\Big]_{-3}^{2} + \left(-\frac{25}{24}y^2 + \frac{125}{12}y\right)\Big]_{2}^{5} = 25.$$

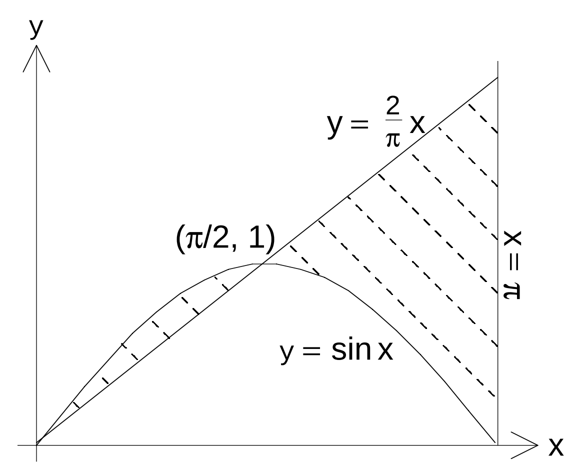
See graph on the right.



40. Since $y = \sin x$ is above the line $y = \frac{2}{\pi}x$ for $0 < x < \frac{\pi}{2}$, and the line is above $y = \sin x$ for $\frac{\pi}{2} < x \le \pi$, $\int_0^{\pi}\left|\sin x - \frac{2}{\pi}x\right|dx =$

$$= \int_0^{\pi/2}\left(\sin x - \frac{2}{\pi}x\right)dx + \int_{\pi/2}^{\pi}\left(\frac{2}{\pi}x - \sin x\right)dx =$$
$$= \left(-\cos x - \frac{1}{\pi}x^2\right)\Big]_0^{\pi/2} + \left(\frac{1}{\pi}x^2 + \cos x\right)\Big]_{\pi/2}^{\pi}$$
$$= \frac{\pi}{2}.$$

This is the area of the region between the curve $y = \sin x$ and the line $y = \frac{2}{\pi}x$, between $x = 0$ and $x = \pi$.
See graph to the right.



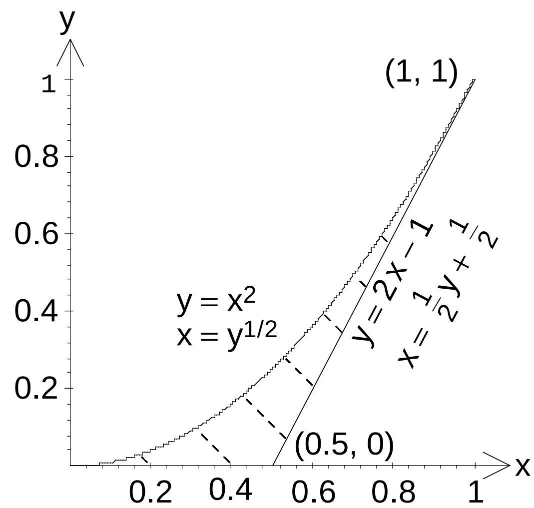
52. The tangent line to the parabola $y = x^2$ at $(1, 1)$ has slope 2 and has equations $y = 2x - 1$ and $x = \frac{1}{2}y + \frac{1}{2}$. The area of the region bounded by the parabola, the tangent line, and the x-axis is

$$\int_0^{0.5} x^2\,dx + \int_{0.5}^{1} (x^2 - 2x + 1)\,dx =$$

$$= \frac{1}{3}x^3\Big]_0^{0.5} + \left(\frac{1}{3}x^3 - x^2 + x\right)\Big]_{0.5}^{1} = \frac{1}{12}.$$

It is easier to find $\int_0^1 \left(\frac{1}{2}y + \frac{1}{2} - y^{1/2}\right) dy =$

$$= \left(\frac{1}{4}y^2 + \frac{1}{2}y - \frac{2}{3}y^{3/2}\right)_0^1 = \frac{1}{12}.$$



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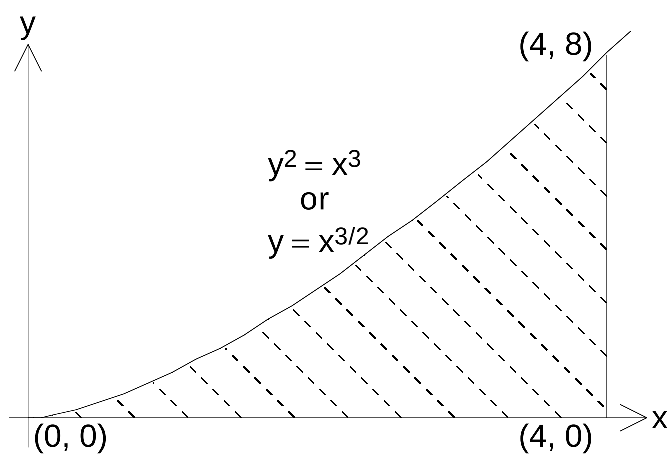
2. $V = \int_0^4 \pi (x^{3/2})^2\,dx =$

$$= \int_0^4 \pi x^3\,dx = \frac{\pi}{4}x^4\Big]_0^4 = 64\pi.$$

See graph to the right.



For Exercise 2

10. $V = \int_0^1 \pi\left[1^2 - (1 - \sqrt{1-y}\,)^2\right] dy =$

$$= \int_0^1 \pi\left[-1 + 2\sqrt{1-y} + y\right] dy =$$

$$= \pi\Big[-y - \frac{4}{3}(1-y)^{3/2} + \frac{1}{2}y^2\Big]\Big]_0^1 =$$

$$= \frac{5}{6}\pi.$$

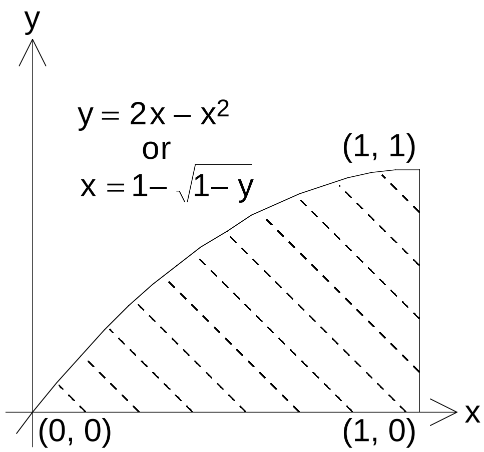
See graph on the next page.

14. $V = \int_0^2 \pi\,[8^2 - (4y)^2]\,dy = \pi\Big[64y - \frac{16}{3}y^3\Big]\Big]_0^2 = \frac{256}{3}\pi.$

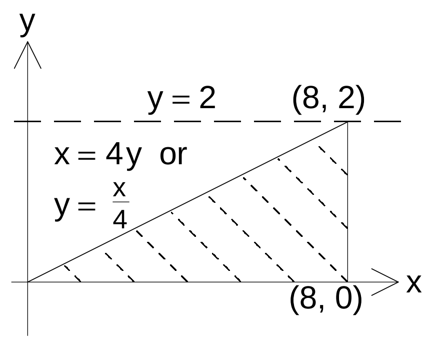
See graph on the next page.

16. $V = \int_0^8 \pi\left[(0-2)^2 - \left(\frac{x}{4} - 2\right)^2\right] dx = \int_0^8 \pi\Big[-\frac{x^2}{16} + x\Big]\,dx = \pi\Big[-\frac{1}{48}x^3 + \frac{1}{2}x^2\Big]\Big]_0^8 = \frac{64}{3}\pi.$

See graph on the next page.

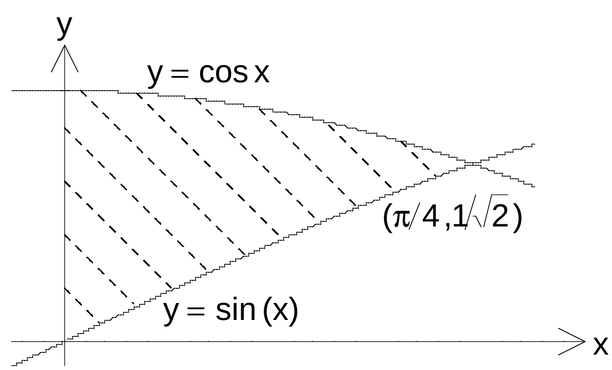


For Exercise 10



For Exercises 14 and 16

30. $V = \int_0^{\pi/4} \pi[\cos^2 x - \sin^2 x]\, dx =$
$= \int_0^{\pi/4} \pi \cos(2x)\, dx = \frac{\pi}{2} \sin(2x)\Big]_0^{\pi/4} = \frac{\pi}{2}.$
See graph to the right.



For Exercise 30

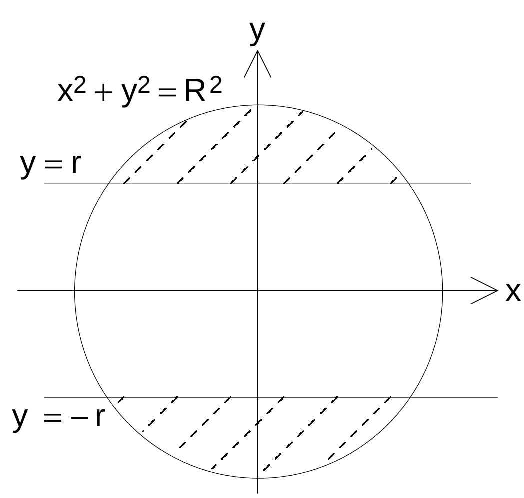
70. If the sphere is formed by rotating the circular disk $x^2 + y^2 \le R^2$ in 3-space about the x-axis and the cylindrical hole has axis of symmetry the x-axis, and if for each value of x we slice through the remaining solid object with a plane perpendicular to the x-axis, we obtain a washer-shaped region with outside radius $\sqrt{R^2 - x^2}$ and inside radius r. Notice that the cylindrical hole emerges from the sphere at $x = \pm\sqrt{R^2 - r^2}$. The portions of the sphere with $|x| \ge \sqrt{R^2 - r^2}$ are part of the removed cylinder. The volume is

$$V = \int_{-(R^2-r^2)^{1/2}}^{(R^2-r^2)^{1/2}} \pi\left[\left(\sqrt{R^2 - x^2}\right)^2 - r^2\right] dx =$$

$$= \int_{-(R^2-r^2)^{1/2}}^{(R^2-r^2)^{1/2}} \pi(R^2 - r^2 - x^2)\, dx =$$

$$= \pi\left[(R^2 - r^2)x - \frac{1}{3}x^3\right]_{-(R^2-r^2)^{1/2}}^{(R^2-r^2)^{1/2}} =$$

$$= \frac{4}{3}\pi(R^2 - r^2)^{3/2}.$$



For Exercise 70

Notice that if $r = 0$ (no hole) then $V = \frac{4}{3}\pi R^3$, the volume of the sphere, and if $r = R$ (nothing left) then $V = 0$.