Quotes

August 21, 2006

Dead Quotes

Aristotle (384 - 322 BC)
Metaphysica, 3-1078b.

> *The mathematical sciences particularly exhibit order, symmetry, and limitation; and these are the greatest forms of the beautiful.*
>
> **Aristotle**

Bellman, Richard (1920 - 1984)
Eye of the hurricane, 1984.

> *[Lefschetz and Einstein] had a running debate for many years. Lefschetz insisted that there was difficult mathematics. Einstein said that there was no difficult mathematics, only stupid mathematicians. I think that the history of mathematics is on the side of Einstein.*
>
> **Richard Bellman**

Bers, Lipman (1914 - 1993)
D Albers, G Alexanderson, C Reid, More Mathematical People, pg. 14

> *Mathematics is an exceedingly cruel profession. You notice that if somebody has a bachelor's degree in chemistry, he describes himself as a chemist. But if somebody has been a professor of mathematics for ten years and you ask him, "Are you a mathematician?" he may say, "I'm trying to be one!" ... The standard is so high, and you never know whether you will be able to hack it. First you are afraid that you won't be able to understand your professors. Then you are afraid that you won't be able to write*

a thesis. When I went to Loewner to ask for a thesis topic, I expected him to grab me by the neck and say, "What makes you think you can write a thesis on mathematics? OUT!!!"

Lipman Bers

pg. 15

I alternate between two attitudes [about attempts to communicate the beauty of mathematics to a wide audience]. Mondays, Wednesdays and Fridays I believe it can be done if we do it properly; Tuesdays, Thursdays and Saturdays I believe it cannot.

Lipman Bers

pg.16

What is the strength of mathematics? What makes mathematics possible? It is symbolic reasoning. It is like "canned thought." You have understood something once. You encode it, and then you go on using it without each time having to think about it. Now there may be people who are totally unable to follow symbolic reasoning—just as I am unable to carry a tune (and yet I do say to myself that I enjoy music). So you must try to explain mathematics without using any symbols. But this may be impossible. Without symbolic reasoning you cannot make a mathematical argument.

Lipman Bers

pg. 16

I think that mathematics is very much like poetry. I think that what makes a good poem—a great poem—is that there is a large amount of thought expressed in very few words. In this sense formulas like

$$e^{\pi i} + 1 = 0$$

or

$$\int_{-\infty}^{+\infty} e^{-x^2}\, dx = \sqrt{\pi}$$

are poems.

Lipman Bers

pg. 19

A working mathematician is always a platonist. It doesn't matter what he says. He may not be a platonist at other times. But I think that in mathematics he always has that feeling of discovery.

Lipman Bers

pg. 20

Usually people do their best work when they are young. And this is probably true of very good mathematicians. But in my case—and I think that most people who know my work will agree—what I did after forty was more interesting and more important than what I did before forty.

Lipman Bers

pg. 20

I disagree with Adler, who wrote (in the **New Yorker***) that there is no point in being a mathematician unless you can be a great mathematician. That's nonsense. Mathematics is like a gothic cathedral. If you can build a little part of it, it is there—forever—in some sense. At least I have the illusion that it is so.*

Lipman Bers

Birkhoff, George David (1884 - 1944)

Musings of the Masters, pg. 103

. . . [a mathematician] holds certain tacit beliefs and attitudes which scarcely ever find their way into the printed page. . . . when he recalls that in the past the most difficult mathematical questions have been ultimately answered, he is inclined to believe with the great German mathematician, Hilbert, that every mathematical fact is provable. Besides all this, he attributes certain values to his results and their mathematical demonstrations; some theories seem important; some proofs are regarded as elegant, others as profound or original, etc. Such somewhat vague ideas illustrate

what I would call mathematical faith. Nearly all the greatest mathematicians have been led to take points of view falling in this broad category, and have attached the deepest significance to them.

George Birkhoff

pg. 110

. . . it is a faith in the uniformity of nature which remains the guiding star of the physicist just as for the mathematician it is a faith in the self-consistency of all mathematical abstractions, although these faiths are more sophisticated than ever before. The minds of both are tinged with an unwavering belief in the supreme importance of their own fields. The mathematician affirms with Descartes, **omnia apud me mathematica fiunt***—with me everything turns into mathematics; by this he means that all permanent forms of thought are mathematical. The physicist on his part is apt to think that there is no reality essentially other than physical reality, so that life itself is finally to be fully described in physical terms.*

George Birkhoff

Boas, Ralph P. Jr. (1913 - 1992)
More Mathematical People, pg. 30-31

Real mathematicians, except for a small number of geniuses, don't do anything **except** *mathematics. . . . Although I am fond of classical music, I never learned to play an instrument, and I am hopelessly unathletic. However, I grew up in the country and summered on Cape Code, so I console myself by being able to do some things that my more cultivated colleagues probably can't. I do, for example, know how to sail a boat, shingle a roof, cut grass with a scythe, and fell a tree so that it will fall where I want it to.*

Ralph Boas

pg. 41

Some years ago, after I had given a talk, somebody said, "You seem to make mathematics sound like so much fun." I was inspired to reply, "If it isn't fun, why do it?" I am proud of the sentiment, even if it is overstated.

Ralph Boas

Bôcher, Maxime (1867 - 1918)

Bulletin of the American Mathematical Society 11 1904, pg. 133.

I like to look at mathematics almost more as an art than as a science; for the activity of the mathematician, constantly creating as he is, guided though not controlled by the external world of senses, bears a resemblance, not fanciful I believe but real, to the activities of an artist, of a painter let us say. Rigorous deductive reasoning on the part of the mathematician may be likened here to technical skill in drawing on the part of the painter. Just as no one can become a good painter without a certain amount of this skill, so no one can become a mathematician without the power to reason accurately up to a certain point. Yet these qualities, fundamental though they are, do not make a painter or mathematician worthy of the name, nor indeed are they the most important factors in the case. Other qualities of a far more subtle sort, chief among which in both cases is imagination, go to the making of a good artist or of a good mathematician.

Maxime Bôcher

pg. 134-135.

... there is what may perhaps be called the method of optimism which leads us either wilfully or instinctively to shut our eyes to the possibility of evil. Thus the optimist who treats a problem in algebra or analytic geometry will say, if he stops to reflect on what he is doing: "I know that I have no right to divide by zero; but there are so many other values which the expression by which I am dividing might have that I will assume that the Evil One has not thrown a zero in my denominator this time."

Maxime Bôcher

Bochner, Salomon (1899 - 1982)

The Role of Mathematics in the Rise of Science

Mathematics is a form of poetry which transcends poetry in that it proclaims a truth; a form of reasoning which transcends reasoning in that it wants to bring about the truth it proclaims; a form of action, of ritual behaviour, which does not find fulfilment in the act but must proclaim and elaborate a poetic form of truth (quoted from "a book on the awakening of intellectuality in Egypt and Mesopotamia"

Salomon Bochner

Cayley, Arthur (1821 - 1895)

J. R. Newman (ed.) The World of Mathematics, New York: Simon and Schuster, 1956. First Volume

Cayley's presidential address in 1883 to the British Association for the Advancement of Science

It is difficult to give an idea of the vast extent of modern mathematics. The word 'extent' is not the right one: I mean extent crowded with beautiful detail—not an extent of mere uniformity such as an objectless plain, but of a tract of beautiful country seen at first in the distance, but which will bear to be rambled through and studied in every detail of hillside and valley, stream, rock, wood, and flower. But, as for everything else, so for a mathematical theory—beauty can be perceived but not explained.

Arthur Cayley

Courant, Richard (1888-1972)

Scientific American 211 September 1964

It becomes the urgent duty of mathematicians, therefore, to meditate about the essence of mathematics, its motivations and goals and the ideas that must bind divergent interests together.

Richard Courant

Methods of Mathematical Physics

Since the seventeenth century, physical intuition has served as a vital source for mathematical porblems and methods. Recent trends and fashions have, however, weakened the connection between mathematics and physics; mathematicians, turning away

from their roots of mathematics in intuition, have concentrated on refinement and emphasized the postulated side of mathematics, and at other times have overlooked the unity of their science with physics and other fields. In many cases, physicists have ceased to appreciate the attitudes of mathematicians. This rift is unquestionably a serious threat to science as a whole; the broad stream of scientific development may split into smaller and smaller rivulets and dry out. It seems therefore important to direct our efforts towards reuniting divergent trends by classifying the common features and interconnections of many distinct and diverse scientific facts.

Richard Courant

What Is Mathematics?: An Elementary Approach to Ideas and Methods By Richard Courant, Ian Stewart, pg. 86

Some distinguished mathematicians have recently advocated the more or less complete banishment from mathematics of all non-constructive proofs. Even if such a program were desirable, it would involve tremendous complications and even the partial destruction of the body of living mathematics. For this reason it is no wonder that the school of "intuitionism", which has adopted this program, has met with strong resistance, and that even the most thoroughgoing intuitionists cannot always live up to their convictions.

Richard Courant

pg. 315

The proofs of Bolzano's and Weierstrass theorems have a decidedly non-constructive character. They do not provide a method for actually finding the location of a zero or the greatest or smallest value of a function with a prescribed degree of precision in a finite number of steps. Only the mere existence, or rather the absurdity of the nonexistence, of the desired value is proved. This is another important instance where the "intuitionists" have raised objections; some have even insisted that such theorems be eliminated from mathematics. The student of mathematics should take this no more seriously than did most of the critics.

Richard Courant

d'Alembert, Jean Le Rond (1717 - 1783)
Discours Preliminaire de L'Encyclopedie, Tome 1, 1967. pp 47 - 48.

Thus metaphysics and mathematics are, among all the sciences that belong to reason, those in which imagination has the greatest role. I beg pardon of those delicate spirits who are detractors of mathematics for saying this... The imagination in a mathematician who creates makes no less difference than in a poet who invents... Of all the great men of antiquity, Archimedes may be the one who most deserves to be placed beside Homer.

Jean d'Alembert

Dantzig, George Bernard (1914 - 2005)
More Mathematical People, pg. 68

If I had known that the problems were not homework but were in fact two famous unsolved problems in statistics, I probably would not have thought positively, would have become discouraged, and would never have solved them.

George Dantzig

pg. 71

The final test of a theory is its capacity to solve the problems which originated it.

George Dantzig

pg. 73

I didn't discover the linear programming model all in a flash. It evolved.

George Dantzig

pg. 77

I had no experience at the time with problems in higher dimensions, and I didn't trust my geometrical intuition. For example, my intuition told me that the procedure would require too many steps wandering from one adjacent vertex to the next. In practice, it takes few steps. In brief, one's intuition in higher dimensional space is not worth a damn!

George Dantzig

pg. 77

I find it amusing that there could be these two very different ways to organize research—one anarchistic, the other dictatorial, and yet both highly efficient. Apparently any form of government can be made to work if the people are motivated enough.

George Dantzig

Dantzig, Tobias (1884 1956)
Number: The Language of Science. pg. 180.

... the progress of mathematics has been most erratic, and that intuition has played a predominant role in it. Distant outposts were acquired before the intermediate territory had been explored, often even before the explorers were aware that there was an intermediate territory. It was the function of intuition to create new forms; it was the acknowledged right of logic to accept or reject these forms, **in whose birth it had no part**.

Tobias Dantzig

pg. 231-232

The mathematician may be compared to a designer of garments, who is utterly oblivious of the creatures whom his garments may fit. To be sure, his art originated in the necessity for clothing such creatures, but this was long ago; to this day a shape will occasionally appear which will fit into the garment as if the garment had been made for it. Then there is no end of surprise and delight.

Tobias Dantzig

pg. 232

Yet delightful though these [mathematical] surprises may be, their discovery is not the moving force behind the creative work of the mathematician. For him, mathematics is the field in which he can best manifest his personality. Mathematics for mathematics' sake! "People have been shocked by this formula," said Poincaré, "and yet it is as good as life for life's sake, if life is but misery."

Tobias Dantzig

pg. 233

The man of science will act as if this world were an absolute whole controlled by laws independent of his own thoughts or acts; but whenever he discovers a law of striking simplicity or one of sweeping universality or one which points to a perfect harmony in the cosmos, he will be wise to wonder what role his mind has played in the discovery, and whether the beautiful image he sees in the pool of eternity reveals the nature of this eternity, or is but a reflection of his own mind.

Tobias Dantzig

pg. 244-245

And so neither in the subjective nor yet in the objective world can we find a criterion for the reality of the number concept, because the first contains no such concept, and the second contains nothing that is free from the concept. How then can we arrive at a criterion? Not by evidence, for the dice of evidence are loaded. Not by logic, for logic has no existence independent of mathematics: it is only one phase of this multiphased necessity that we call mathematics. How then shall mathematical concepts be judged? **They shall not be judged!** *Mathematics is the supreme judge; from its decisions there is no appeal. We cannot change the rules of the game, we cannot ascertain whether the game is fair. We can only study the player at his game; not, however, with the detached attitude of a bystander, for we are watching our own minds at play.*

Tobias Dantzig

Descartes, René (1596 - 1650)
Discours de la Méthode. 1637. Discourse on Method and Related Writings (pg. 47) ISBN: 0140446990

> *If I found any new truths in the sciences, I can say that they follow from, or depend on, five or six principal problems which I succeeded in solving and which I regard as so many battles where the fortunes of war were on my side.*

René Descartes

Dirac, Paul Adrien Maurice (1902 - 1984)
Scientific American, May 1963.

> *I think that there is a moral to this story, namely that it is more important to have beauty in one's equations than to have them fit experiment. If Schrödinger had been more confident of his work, he could have published it some months earlier, and he could have published a more accurate equation. ... It seems that if one is working from the point of view of getting beauty in one's equations, and if one has really a sound insight, one is on a sure line of progress. If there is not complete agreement between the results of one's work and experiment, one should not allow oneself to be too discouraged, because the discrepancy may well be due to minor features that are not properly taken into account and that will get cleared up with further developments of the theory.*

Paul Dirac

Eddington, Sir Arthur Stanley (1882 - 1944)
Number: The Language of Science, pg. 231

> *We have found a strange footprint on the shores of the unknown. We have devised profound theories, one after another, to account for its origin. At last, we have succeeded in reconstructing the creature that made the footprint. And lo! it is our own.*

Sir Arthur Eddington

Einstein, Albert (1879 - 1955)

H Dukas and B Hoffmann (eds.), Albert Einstein : the human side. New glimpses from his archives (Princeton, N.J., 1979). pg. 13

> *I imagine myself becoming a professor in those branches of the natural sciences, choosing the theoretical parts of them. Here are the reasons that have brought me to this plan. Above all, it is my disposition for abstract and mathematical thought, and my lack of imagination and practical ability.*
>
> **Albert Einstein**

pg.17

> *I collect nothing but unanswered correspondence and people who, with justice, are dissatisfied with me. But can it be otherwise with a man possessed? As in my youth, I sit here endlessly and think and calculate, hoping to unearth deep secrets. The so-called Great World, i.e. men's bustle, has less attraction than ever, so that each day I find myself becoming more of a hermit.*
>
> **Albert Einstein**

pg. 18

> *As for the search for truth, I know from my own painful searching, with its many blind alleys, how hard it is to take a reliable step, be it ever so small, towards the understanding of that which is truly significant.*
>
> **Albert Einstein**

pg. 29

> *For the creation of a theory the mere collection of recorded phenomena never suffices—there must always be added a free invention of the human mind that attacks the heart of the matter.*
>
> **Albert Einstein**

pg. 33

Every one who is seriously involved in the pursuit of science becomes convinced that a spirit is manifest in the laws of the Universe—a spirit vastly superior to that of man, and one in the face of which we with our modest powers must feel humble. In this way the pursuit of science leads to a religious feeling of a special sort, which is indeed quite different from the religiosity of someone more naive.

Albert Einstein

pg. 37

What Artistic and Scientific Experience Have in Common

Where the world ceases to be the scene of our personal hopes and wishes, where we face it as free beings admiring, asking, and observing, there we enter the realm of Art and Science. If what is seen and experienced is portrayed in the language of logic, we are engaged in science. If it is communicated through forms whose connections are not accessible to the conscious mind but are recognized intuitively as meaningful, then we are engaged in art. Common to both is that loving devotion to that which transcends personal concerns and volition.

Albert Einstein

pg.78

Music does not **influence** *research work, but both are nourished by the same source of longing, and they complement one another in the release they offer.*

Albert Einstein

Herbert Spencer lecture of 1933, "On the method of theoretical physics" Selected Papers (1945-1980) With Commentary: By Chen Ning Yang pg. 566

But the creative principle resides in mathematics. In a certain sense, therefore, I hold true that pure thought can grasp reality, as the ancients dreamed.

Albert Einstein

The Psychology of Invention in the Mathematical Field. Princeton University Press, Princeton, N. J., 1945.

> *The words or the language, as they are written or spoken, do not seem to play any role in my mechanism of thought. The psychical entities which seem to serve as elements in thought are certain signs and more or less clear images which can be "voluntarily" reproduced and combined.*

Albert Einstein

> *There is, of course, a certain connection between those elements and relevant logical concepts. It is also clear that the desire to arrive finally at logically connected concepts is the emotional basis of this rather vague play with the above mentioned elements. But taken from a psychological viewpoint, this combinatory play seems to be the essential feature in productive thought—before there is any connection with logical construction in words or other kinds of signs which can be communicated to others.*

Albert Einstein

> *The above mentioned elements are, in my case, of visual and some of muscular type. Conventional words or other signs have to be sought for laboriously only in a secondary stage, when the mentioned associative play is sufficiently established and can be reproduced at will.*

Albert Einstein

> *According to what has been said, the play with the mentioned elements is aimed to be analogous to certain logical connections one is searching for.*

Albert Einstein

> *Visual and motor. In a stage when words intervene at all, they are, in my case, purely auditive, but they interfere only in a secondary stage already mentioned.*

Albert Einstein

It seems to me that what you call full consciousness is a limit case which can never be fully accomplished. This seems to me connected with the fact called the narrowness of consciousness (Enge des Bewusstseins)

Albert Einstein

Erdös, Paul (1913 - 1996)

Paul Hoffman, The man who loved only numbers: the story of Paul Erdos and the search for mathematical truth. pg.154

Erdös, who put in 19-hour days proving and conjecturing, denied that he fell asleep during mathematics conferences.
"I wasn't sleeping," **he would say.** *"I was thinking."*

Paul Erdös

pg. 26, pg. 4 (SF)

There's an old debate about whether you create mathematics or just discover it. In other words, are the truths already there, even if we don't yet know them? If you believe in God, the answer is obvious. Mathematical truths are there in the SF's mind, and you just rediscover them. Remember the limericks:
There was a young man who said, 'God,
It has always struck me as odd
That the sycamore tree
simply ceases to be
When there's no one about in the quad. '

'Dear Sir, Your astonishment's odd;
I am always about in the quad:
And that's why the tree
Will continue to be,
Since observed by,
Yours faithfully, God.'

(The SF is the Supreme Fascist, the Number-One Guy Up There, God, who was always tormenting Erdös by

hiding his glasses, stealing his Hungarian passport, or, worse yet, keeping to Himself the elegant solutions to all sorts of intriguing mathematical problems.)

Paul Erdös

Euler, Leonhard (1707 - 1783)

Euler's Opera Omnia, ser.1, vol.2, p.241-253.
Mathematics and Plausible Reasoning, Volume 1: Induction and Analogy in Mathematics, pg. 91

Till now the mathematicians tried in vain to this day to discover some order in the sequence of prime numbers and we have reason to believe that there is some mystery which the human mind shall never penetrate.

Leonhard Euler

Opera Omnia, ser.1, vol.2, p.459
Mathematics and Plausible Reasoning, pg. 3

It will seem not a little paradoxical to ascribe a great importance to observations even in that part of the mathematical sciences which is usually called Pure Mathematics, since the current opinion is that observations are restricted to physical objects that make impression on the senses. As we must refer the numbers to the pure intellect alone, we can hardly understand how observations and quasi-experiments can be of use in investigating the nature of the numbers... The kind of knowledge which is supported only by observations and is not yet proved must be carefully distinguished from the truth; it is gained by induction, as we usually say. Yet we have seen cases in which mere induction led to error. Therefore, we should take great care not to accept as true such properties of the numbers which we have discovered by observatino and which are supported by induction alone. Indeed, we should use such a discovery as an opportunity to investigate more exactly the properties discovered and to prove or disprove them; in both cases we may learn something useful.

Leonhard Euler

Feynman, Richard Philips (1918 - 1988)

*http : //nobelprize.org/nobel$_p$rizes/physics/laureates/*1965/*feynman−lecture.html*

Nobel Lecture, Dec. 11, 1965.

> *We have a habit in writing articles published in scientific journals to make the work as finished as possible, to cover up all the tracks, to not worry about the blind alleys or describe how you had the wrong idea first, and so on. So there isn't any place to publish, in a dignified manner, what you actually did in order to get to do the work.*

Richard Philips Feynman

The Character of Physical Law, MIT Press (1967) pg. 58

> *To those who do not know mathematics it is difficult to get across a real feeling as to the beauty, the deepest beauty, of nature... If you want to learn about nature, to appreciate nature, it is necessary to understand the language that she speaks in.*

Richard Philips Feynman

Galilei, Galileo (1564 - 1642)

Galileo: Father of Modern Science By Rachel Hilliam

Opere Il Saggiatore p. 171.

> *[The universe] cannot be read until we have learnt the language and become familiar with the characters in which it is written. It is written in mathematical language, and the letters are triangles, circles and other geometrical figures, without which means it is humanly impossible to comprehend a single word.*

Galileo Galilei

Galois, Evariste (1811 - 1832)

From the preface to his final manuscript.

Jules Tannery, ed., Manuscrits d'Evariste Galois (Paris: Gauthier-Villars, 1908).

Since the beginning of the century, computational procedures have become so complicated that any progress by those means has become impossible, without the elegance which modern mathematicians have brought to bear on their research, and by means of which the spirit comprehends quickly and in one step a great many computations. It is clear that elegance, so vaunted and so aptly named, can have no other purpose...
Go to the roots, of these calculations! Group the operations. Classify them according to their complexities rather than their appearances! This, I believe, is the mission of future mathematicians. This is the road on which I am embarking in this work.

Galois

Gauss, Johann Carl Friedrich (1777 - 1855)
Musings of the Masters, pg. 89

Finally, two days ago, I succeeded, not on account of my painful efforts, but by the grace of God. Like a sudden flash of lightning, the riddle happened to be solved. I myself cannot say, what was the conducting thread, which connected what I previously knew, with what made my success possible.

Gauss

Mathematical Expeditions, pg.167
C. Schilling and J. Kramer, Wilhem Olbers, Sein Leben und Seine Werke, vol. I, Bertin, 1900/09.

I do admit that the Fermat Theorem as an isolated result is of little interest to me, since it is easy to postulate a lot of such theorems, which one can neither prove nor refute. Nonetheless, it has caused me to return to some old ideas for a **great** *extension of higher arithmetic. Of course, this theory is one of those things where one cannot presuppose to what extent one will succeed in reaching goals looming in the far distance. A lucky star must also preside, and my situation as well as much detracting business do not allow me to indulge in such meditations as during the lucky years 1796-1798, when I formed the main parts of my Disquisitiones Arithmeticae.*
Alas I am convinced, that if **luck** *contributes more than I am allowed to hope for, and I succeed in some of the main steps in*

that theory, then the Fermat theorem will appear in it as one of the least interesting corollaries.

Gauss

J. R. Newman (ed.) The World of Mathematics, New York: Simon and Schuster, 1956. p. 326.

If others would but reflect on mathematical truths as deeply and as continuously as I have, they would make my discoveries.

Gauss

Nelson, L. (1949). Socratic Method and Critical Philosophy. Trans. by T. K. Brown. Yale University Press, New Haven, U.S.A. p. 89
Beveridge, W. I. B. (1950). The Art of Scientific Investigation. London. pg. 145
In A. Arber The Mind and the Eye 1954, pg. 47.

I have had my results for a long time: but I do not yet know how I am to arrive at them.

Gauss

Letter to Bolyai, 1808.
Briefwechsel zwischen Carl Friedrich Gauss und Wolfgang Bolyai

It is not knowledge, but the act of learning, not possession but the act of getting there, which grants the greatest enjoyment. When I have clarified and exhausted a subject, then I turn away from it, in order to go into darkness again; the never-satisfied man is so strange if he has completed a structure, then it is not in order to dwell in it peacefully, but in order to begin another. I imagine the world conqueror must feel thus, who, after one kingdom is scarcely conquered, stretches out his arms for others.

Gauss

Preface to Eisenstein's Mathematische, Abhandlungen (Berlin, 1847), [H. J. S. Smith]

The higher arithmetic presents us with an inexhaustible store of interesting truths, - of thruths too, which are not isolated, but stand in a close internal connection, and between which, as our knowledge increases, we are continually discovering new and sometimes wholly unexpected ties. A great part of its theories derives an additional charm from the peculiarity that important propositions, with the impress of simplicity upon them, are often easily discoverable by induction, and yet are of so profound a character that we cannot find their demonstration till after many vain attempts; and even then, when we do succeed, it is often by some tedious and artificial process, while the simpler methods may long remained concealed.

Gauss

Glaisher, James Whitbread Lee (1848 - 1928)

Presidential Address British Association for the Advancement of Science, Section A, (1890); Nature, Vol.42, p. 467.

In other branches of science, where quick publication seems to be so much desired, there may possibility be some excuse for giving to the world slovenly or ill-digested work, but there is no excuse in mathematics. The form ought to be as perfect as the substance, and the demonstrations as rigorous as those of Euclid. The mathematician has to deal with the most exact facts of Nature, and he should spare no effort to render his interpretation worthy of his subject, and to give to his work its highest degree of perfection. "pauca sed matura" was Gauss' motto.

J. W. Glaisher

The mathematician requires tact and good taste at every step of his work, and he has to learn to trust to his own instinct to distinguish between what is really worthy of his efforts and what is not.

J. W. Glaisher

Hadamard, Jacques Salomon (1865 - 1963)

The Psychology of Invention in the Mathematical Field, Princeton: Princeton University Press, 1945, p112.

Some intervention of intuition issuing from the unconscious is necessary at least to initiate the logical work.

Jacques Hadamard

p39.

The ideas chosen by my unconscious are those which reach my consciousness, and I see that they are those which agree with my aesthetic sense.

Jacques Hadamard

Hardy, Godfrey Harold (1877 - 1947)

Hardy, "Mathematical Proof," *Mind* 38 (1929) 1-12. Reprinted in *Collected Papers of G. H. Hardy*, vol. VII, Clarendon Press, Oxford, 1966-79, pp. 581-606.
Musings of the Masters, pg. 59

I have myself always thought of a mathematician as in the first instance an **observer**, *a man who gazes at a distant range of mountains and notes down his observations. His object is simply to distinguish clearly and notify to others as many different peaks as he can.*

Godfrey H. Hardy

pg. 59

... there is, strictly, no such thing as mathematical proof; that we can... do nothing but **point**; *that proofs are what Littlewood and I call* **gas**, *rhetorical flourishes designed to affect psychology, pictures on the board in the lectures, devices to stimulate the imagination of pupils.*

Godfrey H. Hardy

pg. 59

. . . it is not disputed that mathematics is full of proofs, of undeniable interest and importance, whose purpose is not in the least to secure conviction. Our interest in these proofs depends on their formal and aesthetic properties. Our object is **both** *to exhibit the pattern and to obtain assent. We cannot exhibit the pattern completely, since it is far too elaborate; and we cannot be content with mere assent from a hearer blind to its beauty.*

Godfrey H. Hardy

A Mathematician's Apology, London, Cambridge University Press, 1941.

It is a melancholy experience for a professional mathematician to find himself writing about mathematicians. The function of a mathematician is to do something, to prove new theorems, to add to mathematics, and not to talk about what he or other mathematicians have done.

Godfrey H. Hardy

pg. 113

[In good proofs] there is a very high degree of unexpectedness, combined with inevitability and economy. The argument takes so odd and surprising a form; the weapons used seem so childishly simple when compared with the far-reaching consequences; but there is no escape from the conclusions.

Godfrey H. Hardy

pg.88

A chess problem is genuine mathematics, but it is in some way 'trivial' mathematics. However ingenious and intricate, however original and surprising the moves, there is something essential lacking. Chess problems are unimportant. The best mathematics is **serious** *as well as beautiful.*

Godfrey H. Hardy

pg. 115

I am interested in mathematics only as a creative art.

Godfrey H. Hardy

pg. 85

The mathematician's patterns, like the painter's or the poet's must be beautiful; the ideas, like the colors or the words must fit together in a harmonious way. Beauty is the first test: there is no permanent place in this world for ugly mathematics.

Godfrey H. Hardy

pg. 123

I believe that mathematical reality lies outside us, that our function is to discover or **observe** *it, and that the theorems which we prove, and which we describe grandiloquently as our "creations," are simply the notes of our observations. This view has been held, in one form or another by many philosophers of high reputation, from Plato onwards. ...*

Godfrey H. Hardy

pg. 61

There is no scorn more profound, or on the whole more justifiable, than that of the men who make for the men who explain. Exposition, criticism, appreciation, is work for second-rate minds.

Godfrey H. Hardy

pg. 24

A mathematician, like a painter or poet, is a maker of patterns. If his patterns are more permanent than theirs, it is because they are made with ideas.

Godfrey H. Hardy

pg. 86

The fact is that there are few more "popular" subjects than mathematics. Most people have some appreciation of mathematics, just as most people can enjoy a pleasant tune; and there are probably more people really interested in mathematics than in music. Appearances may suggest the contrary, but there are easy explanations. Music can be used to stimulate mass emotion, while mathematics cannot; and musical incapacity is recognized (no doubt rightly) as mildly discreditable, whereas most people are so frightened of the name of mathematics that they are ready, quite unaffectedly, to exaggerate their own mathematical stupidity.

Godfrey H. Hardy

a 1915 lecture on prime numbers
A Mathematician's Apology pg. 120
The World of Mathematics pg.2138

The theory of Numbers has always been regarded as one of the most obviously useless branches of Pure Mathematics. The accusation is one against which there is no valid defence; and it is never more just than when directed against the parts of the theory which are more particularly concerned with primes. A science is said to be useful if its development tends to accentuate the existing inequalities in the distribution of wealth, or more directly promotes the destruction of human life. The theory of prime numbers satisfies no such criteria. Those who pursue it will, if they are wise, make no attempt to justify their interest in a subject so trivial and so remote, and will console themselves with the thought that the greatest mathematicians of all ages have found it in it a mysterious attraction impossible to resist.

Godfrey H. Hardy

Harish-Chandra (1923 - 1983)

R Langlands, Harish-Chandra, Biographical Memoirs of Fellows of the Royal Society 31 (1985) 198 - 225., pg. 202

In mathematics there is an empty canvas before you which can be filled without reference to external reality.

Harish-Chandra

pg. 206

I have often pondered over the roles of knowledge or experience, on the one hand, and imagination or intuition, on the other, in the process of discovery. I believe that there is a certain fundamental conflict between the two, and knowledge, by advocating caution, tends to inhibit the flight of imagination. Therefore, a certain naivete, unburdened by conventional wisdom, can sometimes be a positive asset.

Harish-Chandra

Heisenberg, Werner Karl (1901 - 1976)

Physics and Beyond (New York 1971) pg. 210

Many people will tell you that an expert is someone who knows a great deal about his subject. To this I would object that no one can ever know very much about any subject. I would much prefer the following definition: An expert is someone who knows some of the worst mistakes that can be made in his subject, and how to avoid them.

Werner Heisenberg

Hermite, Charles (1822 - 1901)

The Mathematical Intelligencer, v. 5, no. 4.
G. Darboux, La vie et l'Oeuvre de Charles Hermite, Revue du mois, 10 January 1906, p. 46.

There exists, if I am not mistaken, an entire world which is the totality of mathematical truths, to which we have access only with our mind, just as a world of physical reality exists, the one like the other independent of ourselves, both of divine creation.

Charles Hermite

The Psychology of Invention in the Mathematical Field. Princeton University Press, Princeton, N. J., 1945.

We are servants rather than masters in mathematics.

Charles Hermite

Hilbert, David (1862 - 1943)
Musings of the Masters, pg. 127

> *. . . there is no unsolvable problem at all. In place of the foolish* **Ignorabimus** *is, in contrast, our slogan:*
> *We must know*
> *We shall know.*

David Hilbert

Opening of his speech to the 1900 Congress in Paris.
"Mathematical Problems," delivered by David Hilbert before the Second International Congress of Mathematicians at Paris in 1900
Bulletin of the American Mathematical Society, vol. 8, 1902, pp.437-445, 478-479

> *Who of us would not be glad to lift the veil behind which the future lies hidden; to cast a glance at the next advances of our science and at the secrets of its development during future centuries? What particular goals will there be toward which the leading mathematical spirits of coming generations will strive? What new methods and new facts will the new centuries disclose in the wide and rich field of mathematical thought?*

David Hilbert

Geometry and the imagination (New York, 1952). pg. iii

> *In mathematics, as in any scientific research, we find two tendencies present. On the one hand, the tendency towards abstraction seeks to crystallize the logical relations inherent in the maze of material that is being studied, and to correlate the material in a systematic and orderly manner. On the other hand, the tendency towards intuitive understanding fosters a more immediate grasp of the objects one studies, a live rapport with them, so to speak, which stresses the concrete meaning of their relations.*

David Hilbert

Musings of the Masters, pg. 121

The appearance of what we call intrinsic harmony is also striking, in a sense other than that used by Leibniz, that it is an embodiment and realization of mathematical thought. ... We can only understand this agreement between nature and thought, between experiment and theory, if we take into consideration the formal component of both sides of nature and our understanding, and the mechanism on which it depends. The mathematical process of analysis gives us, or so it appears, the focus and footings to which matter in the real world, as well as thought in the world of the mind, withdraw and cede control and direction.

David Hilbert

pg. 125

The tool which governs the mediation between theory and practice, between thought and observation is mathematics; it builds the bridge and carries more and more of the load. It thereby happens that the basis of our entire present day culture, in so far as it is based on investigations dealing with nature, can be found in mathematics.

David Hilbert

Ibn Khaldun (1332 - 1406)

The Muqaddimah. An Introduction to History. volume 3, pg.130

Geometry enlightens the intellect and sets one's mind right. All of its proofs are very clear and orderly. It is hardly possible for errors to enter into geometrical reasoning, because it is well arranged and orderly. Thus, the mind that constantly applies itself to geometry is not likely to fall into error. In this convenient way, the person who knows geometry acquires intelligence.

Ibn Khaldun

Jacobi, Carl Gustav Jacob (1804 - 1851)

Number: The Language of Science, pg. 179

To Archimedes came a youth eager for knowledge.
Teach me, O Master, he said, that art divine
Which has rendered so noble a service to the lore of the heavens,

And back of Uranus yet another planet revealed.
Truly, the sage replied, this art is divine as thou sayest,
But divine is was ere it ever the Cosmos explored,
Ere noble service it rendered the lore of the heavens
And back of Uranus yet another planet revealed.
What in the Cosmos thou seest is but the reflection of God,
The God that reigns in Olympus is Number Eternal.

Carl Jacobi

Development of Mathematics in the Nineteenth Century By Felix Klein (pg. 103)
a letter to Dirichlet (July 2, 1830) Crelle's Journal 80:272ff

It is true that Fourier held the opinion that the principal aim of mathematics is public utility and the explanation of natural phenomena; but a philosopher like him should have known that the only end of science is the honor of the human mind, and that, in this respect, a question about numbers is worth as much as a question about the system of the world.

Carl Jacobi

J. R. Newman (ed.) The World of Mathematics, New York: Simon and Schuster, 1956, pg. 1747
Math. Annalen 56:252ff
Carl Gustav Jacob Jacobi By Leo Koenigsberger pp. 131

Mathesis est scientia earum quae per se clara sunt
Mathematics is the science of what is clear by itself.

Carl Jacobi

Jourdain, Philip Edward Bertrand (1879 - 1919)

"The Nature of Mathematics, pp. 4-72 of v. 1 of The World of Mathematics, J. R. Newman (ed.), New York: Simon and Schuster, 1956.

... the process of mathematical discovery is a living and a growing thing.

Philip E. B. Jourdain

Kaplansky, Irving (1917 - 2006)

More Mathematical People, pg. 131

> *Certainly one thing is to look at the first case—the easiest case that you don't understand completely. That general theorem down the road—hopefully you'll get to it by and by. The second piece of advice: do examples. Do a million examples. I think there are shameful cases of people making silly and reckless conjectures just because they didn't take the trouble to look at the first few examples. A well-chosen example can teach you so much.*
>
> **Irving Kaplansky**

> *Sometimes when you work through an example, you suddenly get an insight which you wouldn't have got if you'd just been working abstractly with the hypothesis of your future theorem. I guess both of these are obvious pieces of advice, but they are ignored more often than they should be. ... if the problem is worthwhile, give it a good try. Take months, maybe years if necessary, before you announce to the world, "This is as far as I can go. I'm quitting." It is disgraceful to give up before you have given it a good college try.*
>
> **Irving Kaplansky**

> *The advice of Gauss: publish little but make it good.*
>
> **Irving Kaplansky**

Keynes, John Maynard (1883 - 1946)

Keynes, John Maynard. 1936. The General Theory of Employment, Interest and Money. London: Macmillan. p. xiii

> *The difficulty lies, not in the new ideas, but in escaping the old ones, which ramify, for those brought up as most of us have been, into every corner of our minds.*
>
> **John Keynes**

Khayyam, Omar

Treatise on Demonstration of Problems of Algebra (1070)
Episodes in the Mathematics of Medieval Islam By Lennart Berggren, J L Berggren pp. 123

By the help of God and with His precious assistance I say that algebra is a scientific art. The objects with which it deals are absolute numbers and (geometrical) magnitudes which, though themselves unknown, are related to things which are known, whereby the determination of the unknown quantities is possible. Such a thing is either a quantity or a unique rlation, which is only determined by careful examination. ... What one seaches for in the algebraic art are the relations which lead from the known to the unknown, to discover which is the object of algebra as stated above.

Omar Khayyam

Klein, Felix Christian (1849 - 1925)

Elementary Mathematics from an Advanced Standpoint: Arithmetic, Algebra, Analysis pg. 22

If the activity of a science can be supplied by a machine, that science cannot amount to much, so it is said; and hence it deserves a subordinate place. The answer to such arguments, however, is that the mathematician, even when he is himself operating with numbers and formulas, is by no means an inferior counterpart of the errorless machine, "thoughtless thinker" of Thomae; but rather, he sets for himself his problems with definite, interesting, and valuable ends in view, and carries them to solution in appropriate and original manner. He turns over to the machine only certain operations which recur frequently in the same way, and it is precisely the mathematician – one must not forget this – who invented the machine for his own relief, and who, for his own intelligent ends, designates the tasks which it shall perform.

Felix Klein

Kovalevskaya, Sofia Vasilyevna (1850 - 1891)

Kovalevskaya, Sofia Vasilyevna, *Snya Kovalvsky: Her Recollections of Childhood*, Century (1895), pg. 316

It is impossible to be a mathematician without being a poet in soul.

Sofia Kovalevskaya

Many who have never had an opportunity of knowing any more about mathematics confound it with arithmetic, and consider it an arid science. In reality, however, it is a science which requires a great amount of imagination.

Sofia Kovalevskaya

Landau, Lev Davidovich (1908 - 1968)
related by Erdos in Paul Hoffman's The man who loved only numbers, pg. 279

Wir Mathematiker sind alle ein biszchen meschugge.
We mathematicans are all a little bit crazy.

Lev Landau

Laplace, Pierre-Simon (1749 - 1827)
Introduction to Thorie Analytique des Probabilitis

The theory of probabilities is at bottom nothing but common sense reduced to calculus; it enables us to appreciate with exactness that which accurate minds feel with a sort of instinct for which oft-times they are unable to account.

Pierre-Simon Laplace

Number: The Language of Science, pg. 15

Leibnitz saw in his binary arithmetic the image of Creation. ... He imagined that Unity represented God, and Zero the void; that the Supreme Being drew all beings from the void, just as unity and zero expressed all numbers in his system of numeration. This conception was so pleasing to Leibnitz that he communicated it to the Jesuit, Grimaldi, president of the Chinese tribunal for mathematics, in the hope that this emblem of creation would convert the Emperor of China, who was very fond of the sciences. I mention this merely to show how the prejudices of childhood may cloud the vision even of the greatest men!

Pierre-Simon Laplace

Le Cam, Lucien (1924 - 2000)
More Mathematical People, pg. 174

There are mathematicians who do mathematics simply because they like it—it's a work of art. Some people work very hard on problems that have no relation to anything else.

Lucien Le Cam

pg. 178

Most people think of me as some sort of mathematician dealing in abstract and abstruse problems. This is funny since I have essentially no formal mathematical training.

Lucien Le Cam

Lichnerowicz, André (1915 - 1998)
Musing of the Masters, pg.189

The scholar has devoted his life to research, but it is very rare that, in the advancing years, the spark continues to glow. In a scientific notebook of Pasteur, we find a marginal note: "On the whole, nothing for two years, " and this simple note denotes the anguish, among scholars, of knowing whether the spark has definitely been extinguished or whether the gift of creating science is still available to him.

André Lichnerowicz

pg. 186

A mathematician is first of all an artisan learning by throwing himself against his own spirit, a necessary humility. He dreams and is a bit of an artist... I believe that if my neurophysiologist colleagues took electroencephalograms of mathematicians, they would discover no difference between those of a working mathematician and a composer of music. ... Mathematics carries a form of witness all that the spirit of humans have in common, since mathematics does not depend on a civilization or a culture.

André Lichnerowicz

pg. 186

Mathematics is not interested in the nature of things; it puts 'being' in parentheses; this gives it at the same time its power and ambivalence: it is radically non-ontological.

André Lichnerowicz

Liebniz, Gottfried Wilhelm von (1646 - 1716)
Dantzig, George. Number: The Language of Science, pg. 15

Omnibus ex nihil ducendis sufficit unum. (One suffices to derive all out of nothing.)

Leibniz

New Essays Concerning Human Understanding, IV, XII.

The art of discovering the causes of phenomena, or true hypotheses, is like the art of deciphering, in which one ingenious conjecture often greatly shortens the road.

Leibniz

Littlewood, John Edensor (1885 - 1977)
A Mathematician's Miscellany, Methuen Co. Ltd., 1953. pg. 74

The theory of numbers is particularly liable to the accusation that some of its problems are the wrong sort of questions to ask. I do not myself think the danger is serious; either a reasonable amount of concentration leads to new ideas or methods of obvious interest, or else one just leaves the problem alone. "Perfect numbers" certainly never did any good, but then they never did any particular harm.

J. E. Littlewood

pg.23

I constantly meet people who are doubtful, generally without due reason, about their potential capacity [as mathematicians]. The first test is whether you got anything out of geometry. To have disliked or failed to get on with other [mathematical] subjects need mean nothing; much drill and drudgery is unavoidable before they can get started, and bad teaching can make them unintelligible even to a born mathematician.

J. E. Littlewood

pg.49

In presenting a mathematical argument the great thing is to give the educated reader the chance to catch on at once to the momentary point and take details for granted: his successive mouthfuls should be such as can be swallowed at sight; in case of accidents, or in case he wishes for once to check in detail, he should have only a clearly circumscribed little problem to solve (e.g. to check an identity: two trivialities omitted can add up to an impasse). The unpractised writer, even after the dawn of a conscience, gives him no such chance; before he can spot the point he has to tease his way through a maze of symbols of which not the tiniest suffix can be skipped.

J. E. Littlewood

MacLane, Saunders (1909 - 2005)

American Mathematical Monthly, 1954.

Mathematics, springing up from the soil of basic human experience with numbers and data and space and motion, builds up a far-flung architectural structure composed of theorems which reveal insights into the reasons behind appearances and of concepts which relate totally disparate concrete ideas.

Saunders MacLane

Maxwell, James Clerk (1831 - 1879)

The Scientific Papers of James Clerk Maxwell By James Clerk Maxwell pg.328

Mathematicians may flatter themselves that they possess new ideas which mere human language is as yet unable to express. Let them make the effort to express these ideas in appropriate words without the aid of symbols, and if they succeed they will not only lay us laymen under a lasting obligation, but, we venture to say, they will find themselves very much enlightened during the process, and will even be doubtful whether the ideas as expressed in symbols had ever quite found their way out of the equations into their minds.

James Maxwell

Mittag-Leffler, Magnus G¨sta (1846 - 1927)
Speaking of Science - Page 66
Applied Mathematics: Body and Soul: Derivatives - Page 352

The mathematician's best work is art, a high perfect art, as daring as the most secret dreams of imagination, clear and limpid. Mathematical genius and artistic genius touch one another.

Gösta Mittag-Leffler

Mordell, Louis Joel (1888 - 1972)
Mathematical Expeditions: Chronicles by the Explorers By Reinhard Laubenbacher, David Pengelley pg. 165
L.J. Mordell, Three Lectures on Fermat's Last Theorem, Chelsea Publishing Co., New York, 1962.

Mathematical study and research are very suggestive of mountaineering. Whymper made several efforts before he climbed the Matterhorn in the 1860's and even then it cost the life of four of his party. Now, however, any tourist can be hauled up for a small cost, and perhaps does not appreciate the difficulty of the original ascent. So in mathematics, it may be found hard to realise the great initial difficulty of making a little step which now seems so natural and obvious, and it may not be suprising if such a step has been found and lost again.

L. J. Mordell

Morse, Harold Calvin Marston (1892 - 1977)

Marston Morse, "Mathematics and the Arts," *Bulletin of the Atomic Scientists*, no. 2, (1959) 55-59.
Musings Of The Masters: An Anthology of Mathematical Reflections - Page 85

> *I made the same mistake that artists have made since the time of the Greeks, and placed mathematics alongside of the arts as their* **handmaiden**. *... But mathematics is the* **sister**, *as well as the* **servant** *of the arts and is touched with the same madness and genius.*

Marston Morse

pg. 88

> *Most convincing to me of the spiritual relations between mathematics and music, is my own very personal experience. Composing in an amateurish way, I get exactly the same elevation from a prelude that has come to me at the piano, as I do from a new idea that has come to me in mathematics.*

Marston Morse

pg. 88

> *... discovery in mathematics is not a matter of logic. It is rather the result of mysterious powers which no one understands, and in which the unconscious recognition of beauty must play an important part. Out of an infinity of designs a mathematician chooses one pattern for beauty's sake, and pulls it down to earth, no one knows how. Afterwards the logic of words and of forms sets the pattern right. Only then can one tell someone else. The first pattern remains in the shadows of the mind.*

Marston Morse

pg. 90

> *... what is it that a mathematician wants as an artist. I believe that he wishes merely to understand and to create. He wishes to understand, simply, if possible—but in any case to understand; and to create, beautifully, if possible—but in any case to create.*

Marston Morse

pg. 91

Mathematicians of today are perhaps too exuberant in their desire to build new logical foundations for everything. Forever the foundation and never the cathedral.

Marston Morse

pg. 92

As Dürer knew full well, there is a center and final substance in mathematics whose perfect beauty is rational, but rational "in retrospect." The discovery which comes before, those rare moments which elevate man, and the searchings of the heart which come after are not rational. They are gropings filled with wonder and sometimes sorrow.

Marston Morse

Neumann, John von (1903 - 1957)

"The role of mathematics in the science and in society", address to Princeton Graduate Alumni, June 1954. Cf. Collected Works, 6 Vol., Pergamon, New York, 1961, Vol. VI, pp. 477-490.

But still a large part of mathematics which became useful developed with absolutely no desire to be useful, and in a situation where nobody could possibly know in what area it would become useful: and there were no general indications that it even would be so. ... This is true of all science. Successes were largely due to forgetting completely about what one ultimately wanted, or whether one wanted anything ultimately; in refusing to investigate things which profit, and in relying solely on guidance by criteria of intellectual elegance. ... And I think it extremely instructive to watch the role of science in everyday life, and to note how in this area the principle of **laissez faire** *has led to strange and wonderful results.*

John von Neumann

"The Mathematician" in Robert B. Heywood, *The Works of the Mind*, University of Chicago Press, 1947, pp. 180-187. Reprinted in *John von Neumann, Collected Works*, 6 Vol., Pergamon, New York, 1961, Vol. I, pp. 1-9.
The Musings of Masters, pg. 183

> *As a mathematical discipline travels far from its empirical sources, or still more, if it is second and third generation only indirectly inspired by ideas coming from "reality", it is beset with very grave dangers. It becomes more and more purely aestheticizing, more and more purely* **l'art pour l'art** *... there is a great danger that the subject will develop along the line of least resistance... will separate into a multitude of insignificant branches...*
>
> **John von Neumann**

pg. 172

> *A discussion of the nature of the nature of any intellectual effort is difficult* **per se***—at any rate, more difficult than the mere exercise of that particular intellectual effort. It is harder to understand the mechanism of an airplane, and the theories of the forces which lift and which propel it, than merely to ride in it, to be elevated and transported by it or even to steer it. It is exceptional that one should be able to acquire the understanding of a process without having previously acquired a deep familiarity with running it, with using it, before one has assimilated it in an instinctive and empirical way.*
>
> **John von Neumann**

pg. 183

> *One expects a mathematical theorem or a mathematical theory not only to describe and to classify in a simple and elegant way numerous and* **a priori** *disparate special cases. One also expects "elegance" in its "architectural," structural makeup. Ease in stating the problem, great difficulty in getting hold of it and in all attempts at approaching it, then again some very surprising twist by which the approach, or some part of the approach, becomes easy, etc. Also, if the deductions are lengthy or complicated, there should be some simple general principle involved, which*

"explains" the complications and detours, reduces the apparent arbitrariness to a few simple guiding motivations, etc. These criteria are clearly those of any creative art, and the existence of some underlying empirical, worldly motif in the background—often in a very remote background—overgrown by aestheticizing developments and followed into a multitude of labyrinthine variants, all this is much more akin to the atmosphere of art pure and simple than to that of the empirical sciences.

John von Neumann

Newton, Sir Isaac (1643 - 1727)
D Brewster, Memoirs of Newton.

I know not what I appear to the world, but to myself I seem to have been only like a boy playing on the sea-shore, and diverting myself in now and then finding a smoother pebble or a prettier shell, whilest the great ocean of truth lay all undiscovered before me.

Isaac Newton

Oppenheimer, Julius Robert (1904 - 1967)
"The Tree of Knowledge" in Harper's, 217, 1958.

Today, it is not only that our kings do not know mathematics, but our philosophers do not know mathematics and – to go a step further – our mathematicians do not know mathematics.

J. R. Oppenheimer

Pascal, Blaise (1623 - 1662)
Pensees. 1670.

Reason is the slow and tortuous method by which these who do not know the truth discover it. The heart has its own reason which reason does not know.

Blaise Pascal

pg. 228

We are usually convinced more easily by reasons we have found ourselves than by those which have occurred to others.

Blaise Pascal

Pensees, pg. 3

Those who are accustomed to judge by feeling do not understand the process of reasoning, for they would understand at first sight and are not used to seeking for first principles. And others, on the contrary, who are accustomed to reason from principles, do not at all understand matters of feeling, seeking principles and being unable to see at a glance.

Blaise Pascal

Peirce, Charles Sanders (1839 - 1914)

"The Essence of Mathematics" in J. R. Newman (ed.) The World of Mathematics, New York: Simon and Schuster, 1956. pg. 1781

... mathematics is distinguished from all other sciences except only ethics, in standing in no need of ethics. Every other science, even logic—logic, especially—is in its early stages in danger of evaporating into airy nothingness, degenerating, as the Germans say, into an arachnoid film, spun from the stuff that dreams are made of. There is no such danger for pure mathematics; for that is precisely what mathematics ought to be.

Charles Peirce

pg. 1779

Among the minor, yet striking characteristics of mathematics, may be mentioned the fleshless and skeletal build of its propositions; the peculiar difficulty, complication, and stress of its reasonings; the perfect exactitude of its results; their broad universality; their practical infallibility.

Charles Peirce

Plutarch (46 - 127 BC)

Marcellus, The Parallel Lives (75 ACE)
[*aboutArchimedes* :] In G. Simmons Calculus Gems, New York: McGraw Hill Inc., 1992.

> *... being perpetually charmed by his familiar siren, that is, by his geometry, he neglected to eat and drink and took no care of his person; that he was often carried by force to the baths, and when there he would trace geometrical figures in the ashes of the fire, and with his finger draws lines upon his body when it was anointed with oil, being in a state of great ecstasy and divinely possessed by his science.*

Charles Peirce

Poincaré, Jules Henri (1854 - 1912)

La Valeur de la Science, E. Flammarion, Paris, 1905, Chap. 5, p. 139. a lecture he delivered at the First International Congress of Mathematicians, Zurich, 1897

> *In addition to this [mathematics] provides its disciples with pleasures similar to painting and music. They admire the delicate harmony of the numbers and the forms; they marvel when a new discovery opens up to them an unexpected vista; and does the joy that they feel not have an aesthetic character even if the senses are not involved at all?... For this reason I do not hesitate to say that mathematics deserves to be cultivated for its own sake, and I mean the theories which cannot be applied to physics just as much as the others.*

Henri Poincaré

pg. 147

> *If I may be allowed to continue my comparison with the fine arts, then the pure mathematician who would forget the existence of the outside world could be likened to the painter who knew how to combine colors and forms harmoniously, but who lacked models. His creative power would soon be exhausted.*

Henri Poincaré

Mathematics and Plausible Reasoning, vol.1, pg. 142
Poincaré, La valeur de la science, p. 152

> *The science of physics does not only give us [mathematicians] an opportunity to solve problems, but helps us also to discover the means of solving them, and it does this in two ways: it leads us to anticipate the solution and suggests suitable lines of argument.*

Henri Poincaré

The Foundations of Science: Science and Hypothesis, The Value of Science, Science and Method Page 372

> *What is it indeed that gives us the feeling of elegance in a solution, in a demonstration? It is the harmony of the diverse parts, their symmetry, their happy balance; in a word it is all that introduces order, all that gives unity, that permits us to see clearly and to comprehend at once both the ensemble and the details.*

Henri Poincaré

The Misbehavior of Markets: A Fractal View of Risk, Ruin, and Reward By Benoit B Mandelbrot, Richard L Hudson, pg. 44

> *A scientist worthy of his name, above all a mathematician, experiences in his work the same impression as an artist; his pleasure is as great and of the same nature.*

Henri Poincaré

J. R. Newman (ed.) The World of Mathematics, New York: Simon and Schuster, 1956., pg. 42

> *Thought is only a flash between two long nights, but this flash is everything.*

Henri Poincaré

Psychology of Invention?
Quoted in G Simmons Calculus Gems (New York 1992).

Talk with M. Hermite. He never evokes a concrete image, yet you soon perceive that the more abstract entities are to him like living creatures.

Henri Poincaré

The Psychology of Invention in the Mathematical Field. Princeton University Press, Princeton, N. J., 1945.

Let us represent the future elements of our combinations as something similar to the hooked atoms. During the complete period of rest of the mind, these atoms are immobile. They are, so to speak, hooked to the wall. This complete rest can continue indefinitely without these atoms colliding, and consequently without a single combination being produced by them. By contrast, during a period of apparent rest and unconscious work, several of them are detached from the wall and set into motion. They streak throughout space—I was going to say in the region where they are enclosed, as for example a cloud of gnats, or if you prefer a more learned comparison, as gas molecules in the kinetic theory of gasses. Their mutual collisions could thus produce new combinations.

Henri Poincaré

. . . we entered an omnibus to go some place or other. At the moment when I put my foot on the step, the idea came to me, without anything in my former thoughts seeming to have paved the way for it, that the transformations I had used to define the Fuchsian functions were identical with those of non-Euclidean geometry.

Henri Poincaré

Then I turned my attention to the study of some arithmetic questions apparently without much success and without a suspicion of any connection with my previous researches. Disgusted with my failure, I went to spend a few days at the seaside, and thought of something else. One morning, walking along the bluff, the idea came to me, with just the same characteristics of brevity,

suddenness and immediate certainty, the arithmetic transformations of ternary quadratic forms were identical with those of non-Euclidean geometry.

Henri Poincaré

Henri Poincaré, "Mathematical Invention" (L'invention mathématique), *Enseignement Math.* 10 (1908) 357-371. Translated from the French by the editor.
Musings of the Masters, pg. 20

The genesis of mathematical invention is a problem which should inspire the most lively interest among psychologists. It is a process in which the human mind appears to borrow least from the external world, where it does not act, or does not appear to act, except by itself or upon itself, so that in studying the process of mathematical thought, what we hope to attain is that which is the most essential element in the human mind.

Henri Poincaré

pg. 22

... in repeating an argument that I have learned, I could readily have discovered it myself: or rather, even if this is an illusion, if I were not clever enough to have created it myself, I rediscover it myself, to the extent that I repeat it.

Henri Poincaré

pg. 22

... this intuition of mathematical order, which enables us to perceive harmonies and hidden relations, cannot be present in everyone.

Henri Poincaré

pg. 23

Inventing consists precisely in not constructing combinations that are useless, but in constructing those which are useful and these are a small minority. Inventing mean discerning, means choosing.

Henri Poincaré

pg. 24

For fifteen days, I tried to prove that there could not exist any functions analogous to those I have since called fuchsian. I was then very ignorant; every day I sat at my work table where I passed an hour or two trying a large number of combinations but I could not come to any result. One evening, I drank some black coffee contrary to my custom. I could not sleep and ideas surged in large numbers. I sensed them running into one another to the point where two among them locked, so to speak, to form a stable combination. In the morning, I had established the existence of a class of fuchsian functions...

Henri Poincaré

The Foundations of Science: Science and Hypothesis, The Value of Science, Science and Method-Page 366

The scientist does not study nature because it is useful; he studies it because he delights in it, and he delights in it because it is beautiful. If nature were not beautiful, it would not be worth knowing, and if nature were not worth knowing, life would not be worth living.

Henri Poincaré

Poisson, Siméon Denis (1781 - 1840)
Mathematics Magazine, v. 64, no. 1, Feb. 1991.

Life is good for only two things, discovering mathematics and teaching mathematics.

Siméon Poisson

Pólya, George (1887 - 1985)

Mathematics and Plausible Reasoning: Volume I: Induction and Analogy in Mathematics, pg. vi

You have to guess a mathematical theorem before you prove it.

George Pólya

How to Solve It. Princeton: Princeton University Press. 1945.

Even fairly good students, when they have obtained the solution of the problem and written down neatly the argument, shut their books and look for something else. Doing so, they miss an important and instructive phase of the work. ... A good teacher should understand and impress on his students the view that no problem whatever is completely exhausted. One of the first and foremost duties of the teacher is not to give his students the impression that mathematical problems have little connection with each other, and no connection at all with anything else. We have a natural opportunity to investigate the connections of a problem when looking back at its solution.

George Pólya

The first rule of discovery is to have brains and good luck. The second rule of discovery is to sit tight and wait till you get a bright idea.

George Pólya

A great discovery solves a great problem, but there is a grain of discovery in the solution of any problem. Your problem may be modest, but if it challenges your curiosity and brings into play your inventive faculties, and if you solve it by your own means, you may experience the tension and enjoy the triumph of discovery.

George Pólya

If you have to prove a theorem, do not rush. First of all, understand fully what the theorem says, try to see clearly what it means. Then check the theorem; it could be false. Examine the consequences, verify as many particular instances as are needed to convince yourself of the truth. When you have satisfied yourself that the theorem is true, you can start proving it.

George Pólya

On Plausible Reasoning, in Proceedings of the International Congress of Mathematicians-1950, Vol 1, Providence, RI: American

Why should a mathematician care for plausible reasoning? His science is the only one that can rely on demonstrative reasoning alone. The physicist needs inductive evidence, the lawyer has to rely on circumstantial evidence, the historian on documentary evidence, the economist on statistical evidence. These kinds of evidence may carry strong conviction, attain a high level of plausibility, and justly so, but can never attain the force of a strict demonstration. ... Perhaps it is silly to discuss plausible grounds in mathematical matters. Yet I do not think so. Mathematics has two faces. Presented in finished form, mathematics appears as a purely demonstrative science, but mathematics in the making is sort of an experimental science. A correctly written mathematical paper is supposed to contain strict demonstrations only, but the creative work of the mathematician resembles the creative work of the naturalist: observation, analogy, and conjectural generalizations, or mere guesses, if you prefer to say so, play an essential role in both. A mathematical theorem must be guessed before it is proved. The idea of a demonstration must be guessed before the details are carried through.

George Pólya

Rényi, Alfréd (1921 - 1970)

P. Turán, "The Work of Alfréd Rényi", Matematikai Lapok 21, 1970, pp 199-210.

If I feel unhappy, I do mathematics to become happy. If I am happy, I do mathematics to keep happy.

Alfréd Rényi

Riemann, George Friedrich Bernhard (1826 - 1866)
Hölder, O. [1924] Die Mathematische Methode. Berlin: Springer, pg. 487

> *If only I had the theorems! Then I should find the proofs easily enough.*
>
> **George Riemann**

Robinson, Julia (1919 - 1985)
More Mathematical People, pg. 264

> *I think that I have always had a basic liking for the natural numbers. To me they are the one real thing. We can conceive of a chemistry that is different from ours, or a biology, but we cannot conceive of a different mathematics of numbers. What is proved about numbers will be a fact in any universe.*
>
> **Julia Robinson**

pg. 265

> *I would say that my stubbornness has been to a great extent responsible for whatever success I have had in mathematics. But then it is a common trait among mathematicians.*
>
> **Julia Robinson**

pg. 278

> *I have been told that some people think that I was blind not to see the solution myself when I was so close to it. On the other hand, no one else saw it either. There are lots of things, just lying on the beach as it were, that we don't see until someone else picks one of them up. Then we all see that one.*
>
> **Julia Robinson**

pg. 280

What I really am is a mathematician. Rather than being remembered as the first woman this or that, I would prefer to be remembered, as a mathematician should, simply for the theorems I have proved and the problems I have solved.

Julia Robinson

Russell, Bertrand Arthur William (1872 - 1970)
The World of Mathematics - Page 1581

Calculus required continuity, and continuity was supposed to require the infinitely little; but nobody could discover what the infinitely little might be.

Bertrand Russell

J. R. Newman (ed.) The World of Mathematics, New York: Simon and Schuster, 1956.pg. 4

Pure mathematics consists entirely of such asseverations as that, if such and such a proposition is true of anything, then such and such another proposition is true of that thing. It is essential not to discuss whether the first proposition is really true, and not to mention what the anything is of which it is supposed to be true... If our hypothesis is about anything and not about some one or more particular things, then our deductions constitute mathematics. Thus mathematics may be defined as the subject in whic we never know what we are talking about, nor whether what we are saying is true.

Bertrand Russell

Portraits from Memory. pg. 729 POSTSCRIPT

I wanted certainty in the kind of way in which people want religious faith. I thought that certainty is more likely to be found in mathematics than elsewhere. But I discovered that many mathematical demonstrations, which my teachers expected me to accept, were full of fallacies, and that, if certainty were indeed discoverable in mathematics, it would be in a new field of mathematics, with more solid foundations than those that had hitherto been thought secure. But as the work proceeded, I was continually

reminded of the fable about the elephant and the tortoise. having constructed an elephant upon which the mathematical world could rest, I found the elephant tottering, and proceeded to construct a tortoise to keep the elephant from falling. But the tortoise was no more secure than the elephant, and after some twenty years of very arduous toil, I came to the conclusion that there was nothing more that I could do in the way of making mathematical knowledge indubitable.

Bertrand Russell

Spengler, Oswald Arnold Gottfried (1880 - 1936)

The World of Mathematics edited by James Roy Newman, pg. 2321
The Decline of the West. Ed. Arthur Helps, and Helmut Werner. Trans. Charles F. Atkinson. New York: Oxford UP, 1991.

The mathematic, then, is an art. As such it has its styles and style periods. It is not, as the layman and the philosopher (who is in this matter a layman too) imagine, substantially unalterable, but subject like every art to unnoticed changes form epoch to epoch. The development of the great arts ought never to be treated without an (assuredly not unprofitable) side-glance at contemporary mathematics.

Oswald Spengler

Sylvester, James Joseph (1814 - 1897)

The Collected Mathematical Papers of James Joseph Sylvester By James Joseph Sylvester
"Presidential Address to British Association," The Collected Mathematical Papers of James Joseph Sylvester, vol. 2, University Press, Cambridge, 1908, pp. 650-61.

The world of ideas which [mathematics] discloses or illuminates, the contemplation of divine beauty and order which it induces, the harmonious connexion of its parts, the infinite hierarchy and absolute evidence of the truths with which it is concerned, these, and such like, are the surest grounds of the title of mathematics to human regard, and would remain unimpeached and unimpaired were the plan of the universe unrolled like a map at our feet, and the mind of man qualified to take in the whole scheme of creation at a glance.

J. J. Sylvester

... there is no study in the world which brings into more harmonious action all the faculties of the mind than [mathematics],... or, like this, seems to raise them, by successive steps of initiation, to higher and higher states of conscious intellectual being...

J. J. Sylvester

Turing, Alan Mathison (1912 - 1954)

Alan Turing the Enigma, New York: Simon & Schuster, 1983, p.144.

Mathematical reasoning may be regarded rather schematically as the exercise of a combination of two facilities, which we may call intuition and ingenuity... The activity of the intuition consists in making spontaneous judgements which are not the result of conscious trains of reasonings.

Alan Turing

Ulam, Stanislaw Marcin (1909 - 1984)

The Man who Loved Only Numbers

The first sign of senility is that a man forgets his theorems, the second is that he forgets to zip up, the third sign is that he forgets to zip down.

Stanislaw Ulam

Adventures of a Mathematician, Scribner's, New York, 1976. p. 120

In many cases, mathematics is an escape from reality. The mathematician finds his own monastic niche and happiness in pursuits that are disconnected from external affairs. Some practice it as if using a drug. Chess sometimes plays a similar role. In their unhappiness over the events of this world, some immerse themselves in a kind of self-sufficiency in mathematics. (Some have engaged in it for this reason alone.)

Stanislaw Ulam

Weil, André (1906 - 1998)

The Apprenticeship of a Mathematician. pg. 91

> *Every mathematician worthy of the name has experienced... the state of lucid exaltation in which one thought succeeds another as if miraculously... this feeling may last for hours at a time, even for days. Once you have experienced it, you are eager to repeat it but unable to do it at will, unless perhaps by dogged work...*
>
> **André Weil**

'The future of mathematics', American Mathematical Monthly, May, 1950.

> *If logic is the hygiene of the mathematician, it is not his source of food; the great problem furnish the daily bread on which he thrives.*
>
> **André Weil**

André Weil, "History of Mathematics: Why and How," *Proc. International Congress of Mathematicians, Helsinki, 1978*, vol. 1, Academia Scientiarum Fennica, Helsinki, 1980, pp. 227-36.
The Musings of the Masters, pg. 205

> *... strategy means the art of recognizing the main problems, attacking them at their weak points, setting up future lines of advance. Mathematical strategy is concerned with long-range objectives; it requires a deep understanding of broad trends and of the evolution of ideas over long periods.*
>
> **André Weil**

pg. 207

> *... the ability to recognize mathematical ideas in obscure and inchoate form, and to trace them under the many disguises which they are apt to assume before coming out in full daylight, is most likely to be coupled with a better than average mathematical talent. More than that, it is an essential component of such talent, since in large part the art of discovery consists in getting a firm grasp on the vague ideas which are "in the air," some of them flying all around us, some (to quote Plato) floating around in our own minds.*

André Weil

pg. 210

We know only too well... that one should not invariably assume a mathematician to be fully aware of the work of his predecessors, even when he includes it among his references; which one of us has read all the books he has listed in the bibliographies of his own writings? We know that mathematicians are seldom influenced in their work by philosophical considerations, even when they profess to take them seriously; we know that they have their own way of dealing with foundational matters by an alternation between possibly reckless disregard and the most painful critical attention. Above all, we have learnt the difference between original thinking and the kind of routine reasoning which a mathematician often feels he has to spin out for the record in order to satisfy his pees, or perhaps only to satisfy himself. A tediously laborious proof may be a sign that the writer has been less than felicitous in expressing himself; but more often than not, as we know, it indicates that he has been laboring under limitations which prevented him from translating directly into words or formulas some very simple ideas.

André Weil

Weyl, Hermann Klaus Hugo (1885 - 1955)

Number: The Language of Science, pg. 227

We must learn a new modesty. We have stormed the heavens, but succeeded only in building fog upon fog, a mist which will not support anybody who earnestly desires to stand upon it. What is valid seems so insignificant that it may be seriously doubted whether analysis is at all possible.

Hermann Weyl

Weyl, "The Unity of Knowledge,"
Musings of the Masters, pg. 68

By the mental process of thinking we try to ascertain truth; it is our mind's effort to bring about its own enlightenment by evidence.

Hermann Weyl

At the basis of all knowledge there lies: (1) **Intuition**, *mind's originary act of "seeing" what is given to him. ... (2)* **Understanding and expression**. *Even in Hilbert's formalized mathematics I must understand the directions given me by communication in words for how to handle the symbols and formulas. (3)* **Thinking the possible**. *In science a very stringent form of it is exercised when, by thinking out the possibilities of the mathematical game, we try to make sure that the game will never lead to a contradiction; a much freer form is the imagination by which theories are conceived.*

Hermann Weyl

Unterrichtsblätter fr Mathematik und Naturwissenschaften, 38, 177-188 (1932). Translation by Abe Shenitzer appeared in The American Mathematical Monthly, v. 102, no. 7 (August-September 1995), p. 646.

We are not very pleased when we are forced to accept a mathematical truth by virtue of a complicated chain of formal conclusions and computations, which we traverse blindly, link by link, feeling our way by touch. We want first an overview of the aim and of the road; we want to understand the **idea** *of the proof, the deeper context.*

Hermann Weyl

It is a fact that beautiful general concepts do not drop out of the sky. The truth is that, to begin with, there are definite concrete problems, with all their undivided complexity, and these must be conquered by individuals relying on brute force. Only then come the axiomatizers and conclude that instead of straining to break in the door and bloodying one's hands one should have first constructed a magic key of such and such shape and then the door would have opened quietly, as if by itself. But they can construct the key only because the successful breakthrough enables them to study the lock front and back, from the outside and from the inside. Before we can generalize, formalize and axiomatize there must be mathematical substance. I think that mathematical substance on which we have practiced förmalization in the last few decades is near exhaustion and I predict that the next generation will face mathematics a tough time.

Hermann Weyl

In an obituary by Freeman J. Dyson in Nature, March 10, 1956.

My work has always tried to unite the true with the beautiful and when I had to choose one or the other, I usually chose the beautiful.

Hermann Weyl

Year Book by American Philosophical Society, 1937

The question for the ultimate foundations and the ultimate meaning of mathematics remains open; we do not know in what direction it will find its final solution nor even whether a final objective answer can be expected at all. "Mathematizing" may well be a creative activity of man, like language or music, of primary originality, whose historical decisions defy complete objective rationalization.

Hermann Weyl

Whitehead, Alfred North (1861 - 1947)

J. R. Newman (ed.) The World of Mathematics, New York: Simon and Schuster, 1956., pg.75
Fractals and Chaos: Mandelbrot Set and Beyond By Benoit Mandelbrot pg. 26

Everything of importance has been said before by somebody who did not discover it.

Alfred North Whitehead

Science and the Modern World., pg. 27
The World of Mathematics, pg. 402

The science of Pure Mathematics, in its modern developments, may claim to be the most original creation of the human spirit.

Alfred North Whitehead

Science and the Modern World: Lowell Lectures, 1925, pg. 30
The World of Mathematics, pg. 403

Let us grant that the pursuit of mathematics is a divine madness of the human spirit, a refuge from the goading urgency of contingent happenings.

Alfred North Whitehead

An Introduction to Mathematics. pg. 4
The World of Mathematics, pg. 1134

The progress of science consists in observing interconnections and in showing with a patient ingenuity that the events of this ever-shifting world are but examples of a few general relations, called laws. To see what is general in what is particular, and what is permanent in what is transitory, is the aim of scientific thought.

Alfred North Whitehead

J R Newman, The World of Mathematics (New York 1956).pg.2315

In the study of ideas, it is necessary to remember that insistence on hard-headed clarity issues from sentimental feeling, as it were a mist, cloaking the perplexities of fact. Insistence on clarity at all costs is based on sheer superstition as to the mode in which human intelligence functions. Our reasonings grasp at straws for premises and float on gossamers for deductions.

Alfred North Whitehead

Wiener, Norbert (1894 - 1964)
Ex-Prodigy: My Childhood and Youth. pg. 21

The Advantage is that mathematics is a field in which one's blunders tend to show very clearly and can be corrected or erased with a stroke of the pencil. It is a field which has often been compared with chess, but differs from the latter in that it is only one's best moments that count and not one's worst. A single inattention may lose a chess game, whereas a single successful approach to a problem, among many which have been relegated to the wastebasket, will make a mathematician's reputation.

Norbert Wiener

pg. 212

Mathematics is too arduous and uninviting a field to appeal to those to whom it does not give great rewards. These rewards are of exactly the same character as those of the artist. To see a difficult uncompromising material take living shape and meaning is to be Pygmalion, whether the material is stone or hard, stonelike logic. To see meaning and understanding come where there has been no meaning and no understanding is to share the work of a demiurge. No amount of technical correctness and no amount of labour can replace this creative moment, whether in the life of a mathematician or of a painter or musician. Bound up with it is a judgement of values, quite parallel to the judgement of values that belongs to the painter or the musician. Neither the artist nor the mathematician may be able to tell you what constitutes the difference between a significant piece of work and an inflated trifle; but if he is not able to recognise this in his own heart, he is no artist and no mathematician.

Norbert Wiener

Modern Quotes

Andrews, George

There is a turn on the road along my drive home where I recall realizing instantly how the proof of a particularly troubling theorem had to go.

George Andrews

On the wall of my study is a poster picturing Stephen Leacock (the Canadian humorist) with the caption: "I'm a great believer in luck! I find the harder I work the more I have of it." I would attribute my discovery of Ramanujan's Lost Notebook to luck arising out of hard work.

George Andrews

If you really want to learn something so that it is yours, so to speak, then the moments of insight come along with the hard slogging through computation and reflection. I should note that I do not feel that I create mathematics; I only discover it. So learning is discovery with more signposts provided by someone who has been down this path before.

George Andrews

. . . I really do not have any clear understanding of the creative process. I am just grateful that it happens!

George Andrews

Askey, Dick

Behind beautiful and seemingly important formulas there must be deeper ideas.

Dick Askey

While at a meeting in Philadelphia, I woke up one morning with the right idea. The resulting conjecture was beyond my ability to prove in general, but some special cases of it were done.

Dick Askey

Certain gaps in knowledge needed to be filled and my main role was to feel that these gaps could be filled.

Dick Askey

It requires persistence and ability of high orders which are rare separately.

Dick Askey

Atiyah, Michael Francis

Chance plays a role, but the key thing is to grab the chance. Here insight or intuition are very important.

Michael Atiyah

My discoveries have all arisen indirectly, not by direction. They have come from asking myself questions about something that I think is mysterious or incompletely understood, and trying to get to the bottom of it.

Michael Atiyah

As a student one has time to study in depth and read textbooks. Later this is hardly possible. But one compensates by learning from talking to one's colleagues.

Michael Atiyah

A Eureka experience (I prefer this term) is characterized by suddenly realizing that you have found the missing piece of the jigsaw puzzle. Once found it is obviously right. The depth of the experience depends on how profound the ultimate result is.

Michael Atiyah

Of course I like to follow things generally and this is important in providing a wide base from which to explore.

Michael Atiyah

I usually learn the work of others as a result of my own work—I find I need to look into something carefully.

Michael Atiyah

Berlekamp, Elwyn

My first big discovery was the design of a class of optimal burst-correcting convolutional codes in 1963. I found them by doing lots of computing, studying the patterns of the output, and then proving a reasonably simple formula. However, for reasons I still don't understand, despite my giving many talks on this subject, hardly anyone but me appreciates it.

Elwyn Berlekamp

I think there is a widespread mis-perception that great discoveries are widely recognized as such when they are made. I think that's false. I've served on many prize and awards committees, and I've come to realize that most people grossly underestimate the importance of communicating their results.

Elwyn Berlekamp

In the commercial realm, that's called marketing, advertising, and/or sales... The similarities with the academic world are much greater than most academics are willing to admit. Sometimes the effective salesman gets as much or more credit or an idea than its inventor.

Elwyn Berlekamp

I had been working specifically on trying to prove that LR Hackenbush is NP-hard for several weeks, when most of the insight occurred to me while sitting in church during a long sermon to which I had tuned out.

Elwyn Berlekamp

You won't find anything if you're not prospecting, but if you keep your eyes open, you might stumble on gold when you thought you were looking for silver.

Elwyn Berlekamp

It's somewhat like hunting or fishing. Luck plays a significant part, but your odds are much better if you have some judgement and experience about the regions where you think the big game are likely to be.

Elwyn Berlekamp

I'm not very good at learning by reading. Most of my colleagues and students are far better at that than I am. Most of what I manage to assimilate I acquire by oral conversations and blackboard discussions, almost tutorials, from someone or other who knows it significantly better than I do. Usually I extract enough hints from him or her that I then manage to work it out and reconstruct some version of it myself.

Elwyn Berlekamp

Bombieri, Enrico

I am not an architect or urban planner, rather more of a painter working small paintings depicting what the inspiration leads him.

Enrico Bombieri

My approach to research consists in looking to the mathematical landscape, taking notice of the things I like and judge interesting and of those I don't care about, and then trying to imagine what should be next. If you see a bridge across a river, you try to imagine what lies on the other shore. If you see a mountain pass between two high mountains, you try to imagine what is in the valley you don't see yet but secretly know must be there.

Enrico Bombieri

Thus the first step of discovery consists for me in selecting an area of interest and good problems. How does one decide what is interesting? Usually, this is an instinctive process that takes very little time.

Enrico Bombieri

My attitude towards mathematics is that most of it is lying out there, sometimes in hidden places, like gems encased in a rock. You don't see them on the surface, but you sense that they must be there and you try to imagine where they are hidden. Suddenly, they gleam brightly in your face and you don't know how you stumbled upon them. Maybe they always were in plain view, and we all are blind from time to time.

Enrico Bombieri

That was the initial intuition, and in five minutes I knew it could be done and all the consequences it would entail. I wanted to give the whole thing to Davenport before his departure, so I started working all the details and the main consequences, without stopping. I worked three days and three nights never taking a rest save for eating a little and drinking coffee. When I met Davenport at the station I handed him a manuscript with the title "On the Large Sieve", with everything. This became one of my most well-known papers.

Enrico Bombieri

If I do something, I don't stop right away thinking that I have reached my goal. Rather, I stop and ask myself: What did I really find? What is next? Sometimes this is the first step for real progress.

Enrico Bombieri

I don't see much difference between learning and creating mathematics, both steps are for me inextricably mixed. Reading a paper by another mathematician for me is comparable to a difficult hike in the mountains with the help of a guide, while in creating mathematics you are in a more familiar territory and a guide is not needed, you can follow your own path based on your experience and feeling.

Enrico Bombieri

I no longer stay up all night to work out a problem or read an interesting paper.

Enrico Bombieri

I can compare the AHA! experience to putting together a very complicated puzzle without a blueprint, and suddenly you realize what it should be, and the pieces fall in the proper slot instantly. One does not need to put all the pieces in their proper places. Once you got the idea, the vision where exactly the bridge should be built.I can compare the AHA! experience to putting together a very complicated puzzle without a blueprint, and suddenly you realize what it should be, and the pieces fall in the proper slot instantly. One does not need to put all the pieces in their proper places. Once you got the idea, the vision where exactly the bridge should be built.

Enrico Bombieri

That was the initial intuition, and in five minutes I knew it could be done and all the consequences it would entail. In conclusion, I think that for my best work I need intuition (or illumination, if it comes really suddenly) and also determination in reaching a goal [..] there have been occasions in which ideas came to me almost by chance or almost by themselves. For example, reading a paper one may see almost in a flash how to remove a stumbling block.

Enrico Bombieri

I worked three days and three nights never taking a rest save for eating a little and drinking coffee.

Enrico Bombieri

Brown, George Spencer
The Laws of Form. 1969.

To arrive at the simplest truth, as Newton knew and practiced, requires years of contemplation. Not activity. Not reasoning.

Not calculating. Not busy behaviour of any kind. Not reading. Not talking. Not making an effort. Not thinking. Simply bearing in mind what it is one needs to know. And yet those with the courage to tread this path to real discovery are not only offered practically no guidance on how to do so, they are actively discouraged and have to set abut it in secret, pretending meanwhile to be diligently engaged in the frantic diversions and to conform with the deadening personal opinions which are continually being thrust upon them.

George Spencer Brown

Christodoulou, Demetrios

I do not attribute much to chance; I would attribute all to insight and illumination. In regard to illumination, I would like to add that in my case the best instances have been at night when I am lying in bed, somewhere between consciousness and sleep. It is during these times that I have the greatest power of concentration when all else except my subject lose reality.

Demetrios Christodoulou

I do not try to assimilate the results of others unless I am really interested in their work. In that case however, I will become completely absorbed, as if it is my own research.

Demetrios Christodoulou

My approach to doing mathematics has certainly changed since I was a student, but the most important change occured around 1981 when I was 30 years old. At that time I decided that I shall not work on a number of smaller problems but rather on a few bigger ones. This decision involves certain risk, especially when one is at an early stage of one's carrier, but it is a risk worth taking. The problem which I set out to solve in 1981 I in fact succeeded in solving only in 1999, but I was awarded the Bocher prize for that (and also for the work on the stability of Minkowski).

Demetrios Christodoulou

I have always been attracted by the beauty of geometry and geometric thinking.

Demetrios Christodoulou

Cohen, Paul

More Mathematical People, pg. 53

I had, intuitively, a very strong philosophical feeling about the direction the proof should go; nevertheless, I felt totally frustrated. I was so low at one point that I stopped thinking about the problem for four or five months. Then, during the winter holidays at the end of 1962, I went on a long trip with my wife-to-be, touring the Southwest,, taking in the Grand Canyon and all that. I spent many hours in the car driving, and I began to get a strong feeling of confidence that the thing could be done. I didn't feel that the difficulties were entirely technical. I still felt, however, that there was a kind of philosophical conspiracy which was preventing me from pushing through, so I was still discouraged, and I let the problem lie fallow for some time even after I came back here.

Paul Cohen

pg. 55

I tend to think that you are at your peak around thirty. I'm saying you won't equal it.

Paul Cohen

pg. 58

... once a problem is solved, I get a little bit bored. I guess that's the price you pay for being a problem solver. I am not really interested in problems that don't seem to stand out.

Paul Cohen

pg. 58

I think to some extent that what I would like to do most is to take a problem that looks very complicated and find a solution that is ideal. I think that's what almost every mathematician really wants to do. And I would like to think that the solutions of the great problems in mathematics have that character. Hopefully the Riemann hypothesis, if it's solved, will have that character.

Paul Cohen

Connor

I imagine that a mathematician's brain is similarly engaged in a ceaseless search for striking patterns in the ever-changing stream of ideas, making the mathematician aware of only the patterns found striking by the mathematician's pattern recognition process.

Connor

I have found that exacting work is necessary to become thoroughly acquainted with a problem before there is much hope of getting a solution.

Connor

I have become more suspicious than I used to be about the originality of my ideas.

Connor

I started some 20 years ago to ask students (and colleagues) wanting to tell me some piece of mathematics to tell me directly, perhaps with some gestures, but certainly without the aid of a blackboard. While that can be challenging, it will, if successful, put the problem more firmly and cleanly into the head, hence increases the chances for understanding. I am also now more aware of the fact that explaining problem and progress to someone else is beneficial.

Connor

De Boor, Carl

During the process 'insights' do appear seemingly spontaneously. However, this seems to me to be akin to artists looking at a landscape and being amazed by how interesting this or that view is – an amazement that ignores the fact that their artistic pattern recognition will only make them aware of certain views, namely those that are striking.

Carl de Boor

I learned from Fritz John and Heinz Kreiss (rather late in my mathematical life) that it is a waste of time to spend endless consecutive hours working on a problem, or filling page after page with calculations and scribbles.

Carl de Boor

When, after some considerable, quite non-productive effort, usually while not at all consciously working on the problem, there appears, for no apparent reason, in your brain the answer to that problem, – that's the AHA experience.

Carl de Boor

Certainly it is standard among mathematicians to enjoy these AHA! moments while they last and postpone for a bit (e.g., until the next day) the necessary checking of the insight.

Carl de Boor

Deligne, Pierre Rene

My methods of learning have mainly changed for the worse. I no more take enough time to appropriate a body of work in such a way as to be able to use it as freely as I know how to sit on a chair, without having to think about it.

Pierre Rene Deligne

For me, and in this I am inspired by Grothendieck, the ideal is a proof which is trivial, because it has been preceded by the "correct" definitions.

Pierre Rene Deligne

My main way of proceeding is trying to get an understanding, rather than trying to solve a problem. I first of all want to understand how things stand together, and what symmetries they have—while keeping in the back of my mind questions I care about, so as to be alert if in the whole panorama a way of attack presents itself.

Pierre Rene Deligne

Writing up is painful, but leads to understandings which can be reused.

Pierre Rene Deligne

When reading, my first questions are: what are the tools used, do the definitions make sense, what are the data, what is only supposed to exist. What I try to remember is "picture, possibly conflicting, with caveat, to know what is true, as well as what is provable—and which tools can be used to prove such and such.

Pierre Rene Deligne

A shorter analog: after a partial step, wondering: what does this argument really mean.

Pierre Rene Deligne

Donoho, David L.

Among your greatest works have you ever attempted to discern the origin of the ideas that lead you to your discoveries?
This is too pompous for me to answer.

David L. Donoho

They may become obsessed, reflect very deeply on it etc. etc. This is a prerequisite for being an outstanding researcher. Education has nothing to do with this. It has to do with NOT becoming deeply attached to some topic or topics but developing an all purpose methodology that can be applied to topics cross subject. And working diligently and in an organized away.

David L. Donoho

Obviously you work like hell and once in while you notice something really unexpected.

David L. Donoho

First, you can only do something worthwhile by devoting an embarrassingly extreme amount of time preparing yourself both within a specialty and by reading voraciously and very broadly outside the specialty as well. If they only knew the amount of dedicated concentrated effort involved, most people would be shocked and repelled at the sacrifice involved. (Of course, a few people will have exactly the kind of obsessive personality that drives them to this kind of effort; the rest would find it an unimaginable deprivation). I think this is true even of the greatest mathematicians, and I'll bet it is true of greatness in many other fields as well.

David L. Donoho

So most results are just about choosing the right topic and then working hard. Ahh! who knows. I would not want to discount manic-depression and seasonal affective disorder as important biological issues.

David L. Donoho

That much said, there are people who are just tremendously smarter and quicker than others (typically as a result of being in an obsessive state) and it would be great if they had a solid education to

begin with, so they know about a wide range of fields and can perhaps notice connections to build new bridges or open new fields.

David L. Donoho

Now that I am old (44) I can study quite well and I can learn about fields that I am not really that interested in and don't intend to have a passionate involvement with. When I was younger, I couldn't begin to learn something unless I had an amazing head of steam built up going in, a kind of obsession to know all about it.

David L. Donoho

Doob, Joseph

"Creative process" is for the birds. I just sat around and wondered about what I was interested in.

Joseph Doob

I am afraid I am a bad subject for your investigation. I am not a cosmic thinker.

Joseph Doob

Dyson, Freeman John

"Freeman Dyson: Mathematician, Physicist, and Writer". Interview with Donald J. Albers, The College Mathematics Journal, vol 25, no. 1, January 1994.

The bottom line for mathematicians is that the architecture has to be right. In all the mathematics that I did, the essential point was to find the right architecture. It's like building a bridge. Once the main lines of the structure are right, then the details miraculously fit. The problem is the overall design.

Freeman Dyson

Efron, Brad

I haven't been very interested in my own mental processes.

Brad Efron

I'm afraid I ran out of Aha's a long time ago. They are dangerous after a certain age anyway, leading easily to a guru complex.

Brad Efron

Unless I am very interested, I can hardly assimilate anything these days.

Brad Efron

Inspiration starts things, but only hard work really gets anywhere.

Brad Efron

At first I'm terribly confused, but after awhile I chip away at my wrong ideas until I'm left with an answer. So I think I'm working in the sculptor mode, rather than the inspired painter.

Brad Efron

Faltings, Gerd

One has to search the terrain until one finds an opening (or gives up), and where that is cannot be planned.

Gerd Faltings

However, I do not think that this is quite by chance. Namely one has to spend much time on the subject before one gets inspiration.

Gerd Faltings

Learning work of others of course means following their thoughts, as opposed to thinking for oneself. However I usually try whether I can find my own proofs for the assertions before I dig into their details.

Gerd Faltings

I assume you refer to "AHA-experience" as the feeling experienced when discovering a new insight. Having defined it that way also answers your first question. The intensity is of course regulated by the magnitude of the new insight, and/or the desperateness before. Also I should mention that these experiences are not so uncommon, but many of them do not last long because often the new insight later turns out to be false.

Gerd Faltings

In some sense all insights come suddenly, usually in some impure form which is clarified later.

Gerd Faltings

Also I should mention that these experiences are not so uncommon, but many of them do not last long because often the new insight later turns out to be false.

Gerd Faltings

Feferman, Solomon

Understanding others is often a painful process until one suddenly goes beyond the details and sees whole what's going on. Many things are so difficult or so foreign that that never happens. Teaching other people's mathematics is the best way to achieve understanding.

Solomon Feferman

Usually, specific endeavour in a definite direction, but often the difficulties met in the process are overcome only through insight/inspiration/illumination. But some lines of pursuit came about through the latter, with the sudden idea that it might be possible to do something of a certain kind and/or in a certain way.

Solomon Feferman

Fefferman, Charles

It seems to me that insight is essential, and even the best mathematicians need luck, but if enough people do mathematics, luck will happen sooner or later. Of course, luck alone almost never succeeds. Hard work tilts the odds considerably.

Charles Fefferman

When I read someone else's work, I first just read the statement of the theorem, and try to prove it myself. If I fail (I usually do), then I turn to the paper I'm reading for a hint. After reading a little, I try again, and the process repeats. Nevertheless, it seems to me utterly different from doing reasearch, because in reading, one knows in advance that the desired result is true, and you are allowed to "give up" by reading the paper at hand. When doing research, you don't know whether whatever you are trying to prove is true, and, if you are 100% hopelessly stuck, you still have to keep trying.

Charles Fefferman

I pick my research problems motivated only by my own tastes and interests. As a student, I was influenced by knowing that this or that was a famous problem. I don't know whether that's good or bad, but it's true. Otherwise, of course I am a bit slower and less energetic now than in my student days, but also a bit less ignorant.

Charles Fefferman

Fleming, Wendell

I should add that I still enjoy working on math at age 73. Currently I am collaborating with an economist on a problem of international debt and finance.

Wendell Fleming

Serendipity is very important, but it only works if the ground is suitably prepared. Both chance and insight are quite important. Chance will favour only those who are prepared. Also, one must expect to consider many ideas, which turn out later to be failures.

Wendell Fleming

However, afterward finding a problem and knowledgeable collaborator turned out to be a much more efficient way to get into a new research topic.

Wendell Fleming

Garding, Lars

There is nothing mysterious about the Aha or Eureka. Anybody can have it and has it: you suddenly remember or realize that... (where you left your gloves for instance). Unfortunately, Eureka has got mythical proportions through Archimedes. Anybody can have this experience. Please do not write a chapter with the title Eureka.

Lars Garding

I will argue below that creativity in mathematics does not differ from intellectual creativity in general,. In the first place the creator must have what I call a net. This is a connected collection of facts, results, guesses and so on that the creator keeps in his head and to which he has immediate access.

Lars Garding

To build a net requires time, work, and interest in and love of the material. A creator must live with his net and more or less think of it all the time. The state of the net depends on the brain capacity and quickness of thought of the creator. Thinking about the net can be precise, dreamlike or haphazard or systematic or in pictures or not in pictures. All the ways of the human brain may be useful.

Lars Garding

A net is necessary for a creator but not sufficient. A net offers its owner the possibility to create something new and thus become a true creator. The above applies to engineers, philosophers, physicists, chemists, writers, artists and so on.

Lars Garding

Creativity in mathematics is considered to be mysterious by most people because they cannot imagine what a mathematical net could contain. Most of them even shy away from the opportunity to create a small net from the mathematics taught in the schools. What I have written here is true but unfortunately not scientific.

Lars Garding

Gleason, Andrew M.
More Mathematical People, p. 92

It just came to me out of the blue one day. I suddenly realized that, well, the answer to the question just sits in the fact that there is this almost everywhere differentiability theorem. It has always struck me as so amazing. One half of me had been bouncing around with this theorem a lot and the other half had been doing this problem, and they had never gotten together.

Andrew Gleason

p. 93

I've very much given to being gripped by explicit things. Sometimes little things, sometimes big things. Most of my work has been in response to very explicit, easily stated things. I'm very fond of problems in which somehow an at least very simple sounding hypothesis is sufficient to really pinch something together and make something out of it.

Andrew Gleason

p. 95

It is notoriously difficult to convey a proper impression of the frontiers of mathematics to non-specialists. ... Ultimately the difficulty stems from the fact that mathematics is an easier subject than the other sciences. ... Consequently, many of the important primary problems of the subject—that is, problems which can be understood by an intelligent outsider—have either been solved or carried to a point where an indirect approach is clearly required. The great bulk of pure mathematical research is concerned with secondary, tertiary or higher-order problems, the very statement of which can hardly be understood until one has mastered a great deal of technical mathematics.

Andrew Gleason

p. 97

Often enough mathematicians have been caught off base with some pathological wrinkle in some funny function or something so, you know, you do have to do the proof, but still in the end that much more important function of a proof in my view is to figure out why it works.

Andrew Gleason

Graham, Ronald

Paul Hoffman, The man who loved only numbers pg.56

I look at mathematics pretty globally. It represents the ultimate structure and order. And I associate doing mathematics with control. Jugglers like to be able to control a situation. There's a well-known saying in juggling: 'The trouble is that the balls go

where you throw them.' It's just you. It's not the phases of the moon or someone else's fault. It's like chess. It's all out in the open. Mathematics is really there, for you to discover.

Ronald Graham

Grenander, Ulf

Chance has played a minor role. The main reason for occasional success is perseverance; never give up on a problem, continue day after day, week after week... also when it looks hopeless.

Ulf Grenander

But changing perspective, look at the problem from different angles. Search for similarities with other scientific work. Progress has often occurred at the boundary of two or more disciplines.

Ulf Grenander

My attitude to mathematics has changed radically since I was a student. At that time I though of mathematics as a body of theorems, a static concept. I learned later to look at it as a problem solving activity. The theorems are still important, but perhaps less so nowadays.

Ulf Grenander

I have also learnt that it is important for young mathematicians to work on his/her own problems as early as possible ('mathematics is a young man's game'). Knowledge is needed but not enough, so that book learning should not be emphasized too much.

Ulf Grenander

Halmos, Paul R.

I Want to be a Mathematician, Washington: MAA Spectrum, 1985.

Don't just read it; fight it! Ask your own questions, look for your own examples, discover your own proofs. Is the hypothesis necessary? Is the converse true? What happens in the classical special case? What about the degenerate cases? Where does the proof use the hypothesis?

Paul Halmos

Mathematics is not a deductive science – that's a cliche. When you try to prove a theorem, you don't just list the hypotheses, and then start to reason. What you do is trial and error, experimentation, guesswork.

Paul Halmos

... the source of all great mathematics is the special case, the concrete example. It is frequent in mathematics that every instance of a concept of seemingly great generality is in essence the same as a small and concrete special case.

Paul Halmos

The joy of suddenly learning a former secret and the joy of suddenly discovering a hitherto unknown truth are the same to me – both have the flash of enlightenment, the almost incredibly enhanced vision, and the ecstasy and euphoria of released tension.

Paul Halmos

To be a scholar of mathematics you must be born with talent, insight, concentration, taste, luck, drive and the ability to visualize and guess.

Paul Halmos

Mathematics as a Creative Art, Amer.Scientist 56 (Winter 1968) 380.

Mathematics—this may surprise you or shock you some—is never deductive in its creation. The mathematician at work makes

vague guesses, visualizes broad generalizations, and jumps to unwarranted conclusions. He arranges and rearranges his ideas, and he becomes convinced of their truth long before he can write down a logical proof. The conviction is not likely to come early—it usually comes after many attempts, many failures, many discouragements, many false starts. It often happens that months of work result in the proof that the method of attack they were based on cannot possibly work, and the process of guessing, visualizing and conclusion—jumping begins again... The deductive stage, writing the result down, and writing down its Rogers proof are relatively trivial once the real insight arrives; it is more like the draftsman's work, not the architect's.

Paul Halmos

Kasner, E. and Newman J., Mathematics and the Imagination, New York: Simon and Schuster, 1940.

Mathematics is often erroneously referred to as the science of common sense. Actually, it may transcend common sense and go beyond either imagination or intuition. It has become a very strange and perhaps frightening subject from the ordinary point of view, but anyone who penetrates into it will find a veritable fairyland, a fairyland which is strange, but makes sense, if not common sense.

Kasner

Mathematics is man's own handiwork, subject only to the limitations imposed by the laws of thought.

Kasner

Hochster, Mel

Chance plays some role, not a major one. Insight is very important, while inspiration often occurs only after many, many weeks, months, or even years of hard thought. Therefore relentless tenacity is important. The imaginative use of analogies has played a strong role for me.

Mel Hochster

Huber, Peter J.

I am retired now and indulging in things such as ancient astronomy and Assyriology; I am doing little mathematics or statistics nowadays, and memory is treacherous.

Peter J. Huber

Serendipity is very important, but it only works if the ground is suitably prepared. I guess the reason is that the "straightforward" discoveries are easy to find, even if they may need a little sweat, and thus have been found by people working in the field before you.

Peter J. Huber

Learning work of others of course means following their thoughts, as opposed to thinking oneself. However I usually try whether I can find my own proofs for the assertions before I dig into their details.

Peter J. Huber

If you have an idea, develop it on your own for, say, two months, and only then check whether the results are known. The reasons are: (1) If you try to check earlier, you won't recognize your idea in the disguise under which it appears in the literature. (2) If you read the literature too carefully beforehand, you will be diverted into the train of thought of the other author and stop exactly where he ran into an obstacle.

Peter J. Huber

Perhaps an oil explorations simile is more appropriate. First I would have a promising, brilliant idea (the aha event) which would induce me to drill. But the eureka event ("I found it!") at best would come hours or days later, if and when the oil would begin to gush forth. That the idea had been brilliant and not

merely foolish would be clear only in retrospect, after attempts to verify and confirm it. And later on one tends to suppress and forget foolish ideas because they are embarrassing (but they are indispensable companions to the brilliant ones!).

Peter J. Huber

The difference to Archimedes' age is that in the meantime it has become difficult to have brilliant new ideas whose correctness becomes obvious before the bath water has become uncomfortably cold.

Peter J. Huber

Things are more complicated. If I was stumped by a problem and seemed to walk around a solid smooth, blank wall, then I would consciously stuff the problem into my subconscious, do something entirely different, and hope for some revelation to surface in due time (it often did).

Peter J. Huber

But I should say that my subconscious usually would present a novel way of attack; if it presented a ready-made "solution", it often was quite wrong.

Peter J. Huber

I found that in order to do creative work, I had to be at it without interruption for at least a week at a time.

Peter J. Huber

When I had a successful idea, I could not let loose and worked furiously.

Peter J. Huber

When things had been settled and written up, I felt exhausted and empty, and itched until I had a new promising idea.

Peter J. Huber

Philologists and historians usually accumulate notes on cards before beginning to write linearly. I do not thinks this works so well in mathematics. I worked in spirals.

Peter J. Huber

Kleitman, Dan J.

And now, in imitation of Lewis Carroll:
I have answered four questions and that is enough
... do not give yourself airs.
I cannot listen all day to such stuff,
be off or I'll kick you downstairs!

Dan J. Kleitman

It often does happen that when you are stuck, doing other things and coming back to it later is often the wisest course. Relevant ideas do pop up in your mind when you are taking a shower, and can pop up as well even when you are sleeping (many of these ideas turn out not to work very well), or even when you are driving.

Dan J. Kleitman

Thus while you can turn the problem over in your mind in all ways you can think of and try to use all the methods you can recall or discover to attack it, there is really no standard approach that will solve it for you. At some stage, if you are lucky, the right combination occurs to you, and you are able to check it and use it to put an argument together.

Dan J. Kleitman

You must try and fail by deliberate efforts, and then rely on a sudden inspiration or intuition—or if you prefer—luck.

Dan J. Kleitman

At the level of words, there are really no new ideas. Good results do not come from inventing new words. And even at a somewhat higher level there are not really new ideas. It is extended combinations of ideas that can be new and can solve difficult problems.

Dan J. Kleitman

In working on this problem and in general, mathematicians wander in a fog not knowing what approach or idea will work, or if indeed any idea will, until by good luck, perhaps some novel ideas, perhaps some old approaches, conquer the problem.

Dan J. Kleitman

Mathematicians, in short, are typically somewhat lost and bewildered most of the time that they are working on a problem. Once they find solutions, they also have the task of checking that their ideas really work, and that of writing them up, but these are routine, unless (as often happens) they uncover minor errors and imperfections that produce more fog and require more work.

Dan J. Kleitman

The human mind is not only a computer, but one that has programmed itself. The active parts... get devoted to particular uses in a way that is proportional somehow to the amount of thought that the individual puts into that kind of use. The more a person thinks about some area, the more neurons are devoted to it, and the pathways through them become wider and easier to traverse. Also it seems that tunnelling is possible from one "idea" to another. You look in your mind for something and come back with something else which may, with enough looking and some luck, be just what you need to fill the gap in your argument.

Dan J. Kleitman

What mathematicians write thus bears little resemblance to what they do: they are like people lost in mazes who only describe their escape routes never their travails inside. Of course they come to enjoy being lost. If you are not lost you cannot experience the thrill of finding your way out.

Dan J. Kleitman

Landau, Susan

In Her Own Words: Six Mathematicians Comment on Their Lives and Careers. Notices of the AMS, V. 38, no. 7 (September 1991), p. 704.

There's a touch of the priesthood in the academic world, a sense that a scholar should not be distracted by the mundane tasks of day-to-day living. I used to have great stretches of time to work. Now I have research thoughts while making peanut butter and jelly sandwiches. Sure it's impossible to write down ideas while reading "curious George" to a two-year-old. On the other hand, as my husband was leaving graduate school for his first job, his thesis advisor told him, "You may wonder how a professor gets any research done when one has to teach, advise students, serve on committees, referee papers, write letters of recommendation, interview prospective faculty. Well, I take long showers."

Dan J. Kleitman

Lax, Peter

More Mathematical People, pg. 155

I like to start with some phenomenon, the more striking the better, and then use mathematics to try to understand it. ... There's an aesthetic quality, yes, but if you try to pin that down, you are just begging the question. What is beautiful is purely subjective. Saying something is beautiful may be no different from saying that you have a feeling that something is important. You know, one of the complaints that pure mathematicians have against applied mathematicians is that it is ugly. ... Beauty is in the eye of the beholder. It's a poor guide, aesthetics is. You have to feel that what you are doing is beautiful but, after all, someone used to classical art regards modern art as horrible and ugly.

Peter Lax

Malliavin, Paul

During the war, Hadamard was pennyless as many french at this time, he suffered also deeply from world war I having all his sons dead fighting in the french army at Verdun.
The 1943 book has been in some sense a book written to justify the grant that he received at this time...

Paul Malliavin

When I learn other works I am immediately trying to reconstruct the main results by myself. As Jean Leray said, "Mathematic is not a dead letter which can be store in libraries, it is a living thinking."

Paul Malliavin

[Mathematicians] are more interested in doing mathematics than speaking about it.

Paul Malliavin

Mandelbrot, Benoit B.

It happens that I knew Hadamard personallly since an uncle of mine was his successor at College de France... In addition, please understand that the mathematicians you are "polling" were largely trained in the 1950s or 1960s. Among them I am very atypical and may be the person whose view of mathematics is farthest from the "norms" of yesterday and closest to Hadamard's.

Benoit B. Mandelbrot

My method of work shows no real change, in fact – against every cliché – a steady improvement that may still continue at age 77.

Benoit B. Mandelbrot

When there is a need to fully assimilate something, I must redo everything in my own way.

Benoit B. Mandelbrot

My principal discoveries have arisen "spontaneously." Also, nearly every one was perceived as a change of direction.

Benoit B. Mandelbrot

The topics were first reputed to wander all around. But every so often there was a spurt of after-the-fact "self-organization" and reorganization that affected future progress.

Benoit B. Mandelbrot

As to my creative process, the sole peculiar feature that is identifiable, significant and worthy reporting is the essential role the eye continues to play. Hadamard would have understood this very well.

Benoit B. Mandelbrot

Do I experience feFelings of illumination? Rarely, except in connection with chance, whose offerings I treasure. In my wandering life between concrete fields and problems, chance is continually important in two ways. A chance reading or encounter has often brought an awareness of existing mathematical tools that were new to me and allowed me to return to old problems I was previously obliged to leave aside. In other cases, a chance encounter suggested that old tools could have new uses that helped them expand.

Benoit B. Mandelbrot

My way of reading mathematics has not changed much. I read very quickly, trying first to understand the key points, and only afterwards fill in details if needed.

Benoit B. Mandelbrot

My passion for the history of ideas is boundless and I go to endless lengths, not to hide the influences from which I benefited, but to understand and express them thoroughly. To me, the value of a thought combines its novelty and difficulty with the depth of its roots. The greatest thrill is to add to streams of ideas that already have a long and recognizable past.

Benoit B. Mandelbrot

Marsden, Jerry

One of the most intense experience I had actually turned out to be nonsense. It occurred in a dream in which I really thought that I got insight into a really hard problem. When I got up, I rushed to my desk to think and after an hour realized that it was all gibberish. But it was quite intense. I find, in general, ideas that come to me "in the shower" are more reliable than those that come to me in my sleep (which does not happen very often).

Jerry Marsden

I can be talking to a colleague or my wife or eating breakfast and suddenly, like a voice from the blue, I get told what to do. Hard to explain.

Jerry Marsden

I was not going after it it just happened. This is the opposite of the view of our current grant system, that imagines that one knows in advance what you want to do and then you go out and do it like building a house. Research, really good research is not like that at all.

Jerry Marsden

It is not just chance, but rather inspiration in the presence of lots of surrounding information. The surrounding information is really crucial.

Jerry Marsden

I assimilate the work of others best through personal contact and being able to question them directly

Jerry Marsden

There was little creative things done when I was a student and there was lots of enthusiasm and dedication. Now, being more mature, the creative spirit interestingly has increased and I do less of the "busy work", leaving that to students and postdocs (which is good for them).

Jerry Marsden

McDuff, Dusa

How much of mathematical creation do you attribute to chance, insight, inspiration, or illumination?
This is an impossible question.

Dusa McDuff

The answer came in a flash, unexpectedly, while I was showering the next morning. I saw a picture of the solution, right there, waiting to be described.

Dusa McDuff

In my principle discoveries I have always been thinking hard trying to understand some particular problem. Often it is just a hard slog, I go round arguments time and again seeking for a hole in my reasoning, or for some way to formulate the problem/structures I see. Gradually some insights builds and I get to "know" how things function. But it can also occur while I am doing something else, having a shower, doing the cooking.

Dusa McDuff

My imagination is quite visual (though not as much as some others I know of). Other mathematicians have a feel for algebraic structure; actual equations turn them on – not me...

Dusa McDuff

I try to build/find structure and cohesion in which I am looking at. I think math is a language; one sees things with some internal eye and needs to find a language to express this.

Dusa McDuff

I go round arguments time and again seeking for a hole in my reasoning, or for some way to formulate the problem/structures I see.

Dusa McDuff

I find it very hard to learn the results of others these days unless they are very close to my own research interests. I used to be able to absorb things rather passively, just reading and doing over the chains of ideas. Now I need to work out examples, specific instances of the new ideas to feel that I have any real understanding. Anything one creates oneself is much more immediate and real and so harder to forget.

Dusa McDuff

Mathematical Notices v. 38, no. 3, March 1991, pp. 185-7.

Gel'fand amazed me by talking of mathematics as though it were poetry. He once said about a long paper bristling with formulas that it contained the vague beginnings of an idea which could only hint at and which he had never managed to bring out more clearly. I had always thought of mathematics as being much more straightforward: a formula is a formula, and an algebra is an algebra, but Gel'fand found hedgehogs lurking in the rows of his spectral sequences!

Dusa McDuff

McKean, Henry

Bye and bye I see it, often quite suddenly, and realize that it's all quite simple, as mathematics properly understood must always be.

Henry McKean

I would not call it inspiration, but it is a rapid coming into focus of prior work, both overt and covert. The overt work is much the same as it always was. The covert work (in bed, on the subway, in dreams) is harder now. At the age 71, I no longer have the energy, the stamina to do it properly, so now this part is like a subliminal worry, not very productive, but who knows? Perhaps it still helps in its way.

Henry McKean

I don't believe that any true progress arises spontaneously. I believe it is always the result of lots of hard work, covert or overt, with the understanding that old work will sometimes come into a new focus so that you get something, if not for free, then at no extra cost. Such "inspiration" is the outcome of covert work and so can be surprising, but the work has to have been done, even if invisibly.

Henry McKean

My usual mode is to jump in and compute (I cannot really think without a pen in hand). Then having computed fast and probably wrong, I find that this particular calculation would not have done what I wanted anyhow, so I throw it out and start over.

Henry McKean

Sometimes I simply repeat what are at bottom the same stupidities for weeks, and though this looks useless on the face of it, I get familiar with the question and learn a few tricks. Of course I know already what I want to come out, mostly by analogy with old things of my own or others, and I'm looking for the mathematical mechanism that makes it work.

Henry McKean

Porzner says: "Gelfand cannot solve difficult problems. He only solves simple problems".

Henry McKean

McMullen, Curtis

However I would like to mention that I find it almost impossible to have a creative thought while sitting at a desk. To the extent I "discover" things it is almost always while walking or pacing. Of course I can "work things out" at a desk, or on the computer, but to really "turn things over in my mind" I have to walk around. Similarly I think Hadamard mentions somewhere that he "thinks with his legs".

Curtis McMullen

Meyer, Yves

My style is mostly original. I loved to attack problems which seemed out of reach. You then have to build everything from scratch. It is like exploring a new continent. A feeling of wilderness.

Yves Meyer

Somehow the intense relaxation of the journey let my brain function correctly. Indeed I sometimes feel that my own intellectual tension prevents the brain from functioning adequately.

Yves Meyer

To my opinion, mathematical intense creativity is the output of a kind of long crisis where one accumulates enough intellectual tension. This tension is hopefully liberated by a discovery that seems to happen within a few seconds of time. It is a kind of vision (internal vision). A kind of mystical experience which produces a remarkable happiness.

Yves Meyer

Morawetz, Cathleen Synge
More Mathematical People, pg. 238

Ah, there's no excitement to beat the excitement of proving a theorem! Until you find out the next day that it's wrong. ... I'll tell you, though, there is something about being a mathematician that is extremely difficult. One of my children put it this way: It's that you're on stage all the time. You can't fake or shift the subject of conversation and so on. That's very demanding of people.

Yves Meyer

pg. 238

I find that I may have emphasized the need to escape from the devils of mathematics to embark on the pleasures of the real world. But it works both ways, and sometimes the devils of the real world drive one into the pleasures of studying mathematics.

Yves Meyer

Mumford, David

My belief is that you pick up ideas from many many sources, books glanced at, bits of lectures by others and they mull around in your unconscious. It is important to want desparately to understand something, to wonder about it when going to sleep and waking up, etc. At some point, things click and you say, "there's something". Usually you don't know where the clues came from (and this is the source of many priority conflicts – you may have picked up an idea from x, but you forget this). Other times, there's plain hard work involved: trying to see all the ways various things might combine, seeking the right estimate or substitution, working out the algebra.

David Mumford

Could you comment on the differences in the manner in which you work when you are trying to assimilate the result of others (learning mathematics) as compared to when you are indulging in personal research (creating mathematics)?

"indulging": an odd verb to use. You need to fit other people's work into your own framework, that's the challenge in reading others. Personally, I usually need an example, which is like a stepladder, boosting me up.

David Mumford

Papanicolaou, George

I often find that a whole area of physics or applications becomes transparent to me almost immediately as soon as I understand the mathematical structure that it has, the hard mathematical questions that it poses.

George Papanicolaou

I look at papers only after I have had some overall idea of a problem and then I do not look at details.

George Papanicolaou

Now I hardly read books in the fields I know, and often I do not like the books I read (for example in financial math or imaging and random media) because very few of them really contribute to the subject.

George Papanicolaou

When an idea comes up that solves a hard problem that has been with you for a while you just know it is IT.

George Papanicolaou

In the three or four cases where a clear advance was made the degree to which the idea worked out as hoped for is a measure of its importance and the satisfaction that it gives. Sometimes it is the simplicity of the idea or the ultimate simplicity of the results it gives.

George Papanicolaou

Sometimes after thinking about a problem a complete solution comes out as if it had been worked out in detail before. I am not sure how this happens, perhaps because some methods and tools do become second nature to us after a while.

George Papanicolaou

Peskin, Charles

I'm convinced that I do my best work while asleep. The evidence for this is that I often wake up with the solution to a problem, or at least with a clear idea of how to proceed to solve it.

Charles Peskin

Rudin, Mary Ellen
More Mathematical People, pg. 301

I say that there are lots of problems in mathematics that are interesting but have not been solved, and every time you solve one you think up a new one. Mathematics, therefore, is something that expands rather than contracts. And I tell that these questions are interesting just because you've followed a line of reasoning up to a certain point and the next natural thing to ask is what you're looking at. But that's really not explaining to them what kinds of things might be interesting to me. Sometimes that's pretty hard to explain—even to another mathematician.

Mary Ellen Rudin

pg. 301

Mathematics is obviously something that women should be able to do very well. It's very intuitive. You don't need a lot of machinery, and you don't need a lot of physical strength. You just need stamina, and women often have a great deal of stamina.

Mary Ellen Rudin

Shepp, Larry

One of my early mentors once asked me whether I ever got a great idea while having sex. I said that I had not had observed this phenomenon, whereupon he said that he also never had observed this happening either. We had a good laugh over it.

Larry Shepp

How much did chance enter? It's a good question - it seems I got lucky quite often. Somehow I knew what I needed to know. It's a scary feeling. Sometimes it's hard to not believe in a greater presence, though the evidence is far from scientific. It has been said and I am paraphrasing, that "smart people are the ones who get lucky".

Larry Shepp

My work now at age 70 is a bit different; I still try to solve problems but I have moved to a different class of problems. I try to avoid getting bogged down with ones that need careful calculations which are more difficult for me to do now. I like to believe that my experience helps me to avoid problems that are too difficult, though it's hard to know whether I am simply "playing it safe". I have a long list of problems that I have been stuck on for many years and which may be too hard. But sometimes revisiting them produces a solution. It is surely so that the most important thing for a scientist is to find "the right question".

Larry Shepp

Ideas come at strange times, once when I was unlocking the door to my home—I suddenly saw the entire solution to a problem I had been laboring over for a month or more. In another case I was having lunch with someone and jumped up off my chair more or less shouting, "I got it". In each of these cases, I was not aware that I was actually working on the problem but I suppose I must have been doing so somewhere in the back of my mind. Such "Eureka" moments produce a wonderful thrill, but is it innovation? One gets the same thrill when one solves a problem as a puzzle, even when one sees the idea in a chess or bridge problem or even a crossword answer. These are clearly not truly innovative since the solution is not at all original, except to oneself.

Larry Shepp

I'm not so loyal to mathematics as most mathematicians are. ... In many ways I'm different from most people. I"m not loyal to my subject.

Stephen Smale

pg. 320

Beauty is very integrated with rarity. ... Beauty is connected so much with innovation and priority ... [In mathematics], it has to be something special to make it beautiful. If it's just ordinary, it's not beautiful.

Stephen Smale

Sternberg, Shlomo

More Mathematical People, pg. 94

Mathematics is the science of order and mathematicians seek to identify instances of order and to formulate and understand concepts that enable us to perceive order in complicated situations.

Shlomo Sternberg

Stewart, Ian

Does God Play Dice? The Mathematics of Chaos. Blackwell, Cambridge, MA, 1989, p. 39.

The successes of the differential equation paradigm were impressive and extensive. Many problems, including basic and important ones, led to equations that could be solved. A process of self-selection set in, whereby equations that could not be solved were automatically of less interest than those that could.

Ian Stewart

Taylor, Jean

Insight to me is essential. I need to have an idea where things are likely to go. I think I learn the most, however, when my insight turns out to be lacking in some way, and I am surprised at the results. Again, it takes a lot of rigorous effort to turn an insight into something that could be called mathematics.

Jean Taylor

I think I just work very hard when I'm being productive. I seem to go into some kind of a trance; when someone talks to me when I'm in that state, I have to struggle to climb out and pay attention, and then it can take a while to get back into the groove. There are times when I just can't seem to have all the neurons firing that are needed for the problem, and there is no way to marshal them; I just have to wait until the next day.

Jean Taylor

People can have insights and inspirations, but if someone doesn't do the work of getting all the details down, it cannot become part of mathematics. Thus the interesting thing to me isn't necessarily the origins but the intense concentration that mathematics requires.

Jean Taylor

Thurston, William

More Mathematical People, pg. 326

It's the same mathematics, whether it's applied or not. It's just important to think of a problem in a general context. I really think all mathematics is applicable.

William Thurston

pg. 332

I think of myself as learning the outskirts of mathematics. I think mathematics is a vast territory. The outskirts of mathematics are the outskirts of mathematical civilization. Then are certain subjects that people learn about and gather together. Then there is a sort of inevitable development in those fields. You get to the point where a certain theorem is bound to be proved, independent of any particular individual, because it is just in the path of development. I enjoy trying to find mathematical topics that people haven't thought to think about. Then I work there. I like elbow room.

William Thurston

pg. 333

... basic research in mathematics is detached from reality in its motivation.

William Thurston

pg. 333

It's very time-consuming to write things down. ... I worry a lot. I try to avoid thinking about new subjects. But I still get behind. It's too easy to be distracted. There are too many interruptions.

William Thurston

pg. 334

There is great power in truth and sincerity. The mathematics community has tremendous reserves of human potential energy. If we are lean and hungry, we are likely to use our energy. If we are honest, it is likely to be effective...

William Thurston

pg. 335

... mathematicians enjoy being in the ivory tower. I think that most mathematicians love mathematics for mathematics' sake. They really do like the feeling of being in an ivory tower. For the most part they are not motivated by applications. But I believe that, whatever their personal motivation is for doing mathematics, in most cases the mathematics they generate will ultimately have significant applications. The important thing is to do the mathematics. But, of course, it's important to have people thinking about applications too.

William Thurston

pg. 335

There are mathematicians, and then there's the rest of the world, and not much interaction between the two.

William Thurston

pg. 335

The inner force that drives mathematicians isn't to look for applications; it's to understand the structure and inner beauty of mathematics.

William Thurston

pg. 336

Personally I like to see lots of relations between lots of different things. I really enjoy that kind of integration you can have when you take very particular nitty-gritty questions and tie them together in very abstract theories.

William Thurston

pg. 337

Very little [of mathematics] is easily accessible. But I think a lot more of it can be explained so that a lot more people understand it. On the level we're talking about. I like to try to make mathematics easy, not to make it hard. I think there is a tendency among mathematicians to try to make it hard. I try to combat that when I see people wrap up their mathematics in formal fancy theories that make it less accessible.

William Thurston

pg. 337

I think that vision is somehow distracting to the spatial sense, because we have a spatial sense that is more than just vision. People associate it with vision, but it's not the same. ... Sometimes pictures can get in the way. Sometimes one can evoke better pictures in one's head just by words. The spatial image is important, but it's what's in the head that counts.

William Thurston

pg. 340

Part of the reason [mathematicians] don't use computers is lack of money.

William Thurston

pg. 340-341

Why do we try to prove things anyway? I think because we want to understand them. We also want a sense of certainty. Mathematics is a very deep field. It's results are stacked very high, and they depend on each other a lot. You build a tower of blocks but if one block is a bit wobbly, you cant build the tower very high before it will fall over. So I think mathematicians are concerned about rigor, which gives us certainty. That's one reason that we concentrate so much more on proof than do other scientists. But I also think proofs are so that we can understand. I guess I like explanations rather than step-by-step rigorous demonstrations.

William Thurston

pg. 341

. . . people use these various things that have been proved in logic to try to make conclusions about mathematics. Some mathematicians then begin to take a sort of solipsistic attitude and say that we can make mathematics up, however we like. It's not determined. I guess I believe in a sort of mathematical reality, but it's a reality that we can't hope to understand completely. It's hard to justify mathematical reality solely on the basis of formal reasoning.

William Thurston

Weinstein, Alan

Most of what I "discover" in my dreams turns out to be nonsense.

Alan Weinstein

It's harder now for me to follow long complicated arguments. I don't read as much.

Alan Weinstein

Zeeman, Erik Christopher
Catastrophe Theory, 1977.

Technical skill is mastery of complexity while creativity is mastery of simplicity.

Christopher Zeeman

C Arnot, Interview with Professor Sir Christopher Zeeman, Warwick The Magazine (6) (Spring 2005), 20-21.

When asked whether he regards mathematics as an art or a science, he replied:
Both. Sometimes you invent it; sometimes you discover it. You have to invent maths to get a solution to a problem but, in the process, I often discover a whole lot more which I didn't expect.

Christopher Zeeman