MATHEMATICS
ON MAINSTREET

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Preamble

• We live, both philosophically and technologically, in a mathematical world. More so than most of us care to admit.

• Our faith in science is in large part based on the "unreasonable efficacy of mathematics" that has been apparent at least since the time of Newton.

• Mathematics has also been arcane and inaccessible.

• There is some hope that, with computers, this will change....

Predicting the Past

"It would appear that we have reached the limits of what it is possible to achieve with computer technology, although one should be careful with such statements, as they tend to sound pretty silly in 5 years."

John Von Neumann (ca. 1949)

★★★★★★

- Von Neumann was in good company....

"Fooling around with alternating current is just a waste of time. Nobody will use it, ever."

Thomas Edison

"I think there is a world market for about five computers."

T.J. Watson, CEO, IBM, 1947

"I have traveled the length and breadth of this country and talked with the best people, and I can assure you that data processing is a fad that won't last out the year."

Chief business ed., Prentice Hall , 1957

• Computers are the legitimate progeny of mathematicians. They were invented, analyzed and found wanting long before any were ever built.

• At least ten years before the construction of anything really resembling a digital computer, mathematicians had proved the limitations of computers.

• Mathematicians spawned computers but then largely ignore them. They rarely if ever helped - and if they did it was in an aesthetically unacceptable way.

"Beauty is the first test; there is no permanent place in the world for ugly mathematics." G.H. Hardy

"Computers are useless. They can only give you answers."

Pablo Picasso (1881-1973)

★★★★★★

- Picasso is wrong and this is the key to why the face of pure mathematics will change completely over the next 50 years.

- Consider $P \neq NP$

- Consider Factoring Integers.

- Consider Multiplication.

- Consider Fractals.

Factoring large Integers

•This is a problem that was answered 70 years ago.

• RSA-129 was factored in 1994.

114381625757888867669235779976146612010218296721242362562561842935706935245733897830597123 563958705058989075147599290026879543541

= 3490529510847650949147849619903898133417764638493387843990820577 * 32769132993266709549961988190834461413177642967992942539798288533

• This is all intimately tied to the Riemann Hypothesis. And, surprisingly enough, there is a lot of money riding on it.

The following 200 digit number is not a prime. The challenge is to find the two factors.

RSA-200 = 27997833911221327

87082946763872260162107044678

69554285375600099293261284001

07609345671052955360856061822

35191095136578863710595448200

65767750985805576135790987349

50144178863178946295187237869

221823983

The Safest Prediction of All

★★★★★★

" The undergraduate curriculum will be completely rewritten"

★★★★★★

- and it is about time!

- There will be no such thing as a non-computationally adept scientist or mathematician

- Maple already outperforms the average undergraduate in many courses.

On The Experimental Nature of Math.

An Example:

$$\sum_{k=1}^{\infty}\left(1+\frac{1}{2}+\cdots+\frac{1}{k}\right)^{4}(k+1)^{-5}$$

$$=-\frac{29}{2}\zeta(9)+\frac{37}{2}\zeta(4)\zeta(5)$$

$$+\frac{33}{4}\zeta(3)\zeta(6)-\frac{8}{3}\zeta^{3}(3)-7\zeta(2)\zeta(7)$$

• It is true - it is not (yet) proven.

• The proof of the pudding is in the eating.

Reliability and Necessity

“Imagine if every Thursday your shoes exploded if you tied them the usual way. This happens to us all the time with computers, and nobody thinks of complaining.”

Jeff Raskin, interviewed in Doctor Dobb’s Journal

★★★★★★

“Computers make it easier to do a lot of things, but most of the things they make it easier to do don’t need to be done.”

Andy Rooney

The 400 Billionth Binary Digit of Pi is 1.

- 9C381872D27596F81D0E48B95A6C46 (actually 100 Billionth Hex)

- by Fabrice Bellard using idle time of nine workstations

- Amazing that it can be done

- Requires the following (experimentally uncovered) identity of Bailey, Plouffe and PB. This is really the interesting part of the story.

- Remember the Pentium (and Brun's constant)

Theorem.

$$\pi = \sum_{i=0}^{\infty} \frac{1}{16^i}\left(\frac{4}{8i+1} - \frac{2}{8i+4} - \frac{1}{8i+5} - \frac{1}{8i+6}\right).$$

Proof. This is equivalent to:

$$\pi = \int_0^{1/\sqrt{2}} \frac{4\sqrt{2} - 8x^3 - 4\sqrt{2}x^4 - 8x^5}{1 - x^8} dx.$$

which on substituting $y := \sqrt{2}x$ becomes

$$\pi = \int_0^1 \frac{16\,y - 16}{y^4 - 2\,y^3 + 4\,y - 4} dy\,.$$

The equivalence follows from the identity

$$\int_0^{1/\sqrt{2}} \frac{x^{k-1}}{1 - x^8}\,dx = \int_0^{1/\sqrt{2}} \sum_{i=0}^{\infty} x^{k-1+8i}\,dx$$

$$= \frac{1}{\sqrt{2}^k} \sum_{i=0}^{\infty} \frac{1}{16^i(8i+k)}$$

□

"Progress is measured funeral by funeral"