What is Organic Mathematics?

Jonathan M. Borwein, Peter B. Borwein, Robert M. Corless,
Loki Jörgenson, and Nathalie Sinclair

April 24, 1996

Contents

1 The *Proceedings* of the Organic Mathematics Workshop

On December 12-14, 1995, the Organic Mathematics Workshop was held at Simon Fraser University, Vancouver, Canada. It featured fifteen invited speakers from around the world, each expert in the field of experimental mathematics. Following their direct, hands-on contributions on the final day of the workshop, the on-line compilation of their contributed research publications was constructed. On April 19, 1996, the *Proceedings of the Organics Mathematics Workshop*[1] was officially released to the Internet via the World Wide Web.

2 Life in the Fast Lane

Mathematics is more important now than it has ever been. More mathematics is being done both inside and outside of universities. Admittedly not all of it is *called* mathematics—among other things, it might be called robotics, or financial analysis, or operations control, or engineering, —but when we look closely there is no doubt that we are living in the truest Golden Age for mathematics to date.

There are indications that this Golden Age is giving way to yet another due to the influence of technology. Mathematics has fundamentally affected technology, most notably computers. It is now clear that the converse will also be true. Technology, usefully falling into five major categories, has already changed mathematics dramatically, and the pace of change is accelerating. The categories are

1. **Communications (between researchers)**: Ordinary mail, phone, fax, e-mail, and now the Web make the synergy of group effort progressively more effective. This development is at least as much software as hardware—there is a (more or less) standard mode of communicating mathematical papers amongst ourselves, based on Donald E. Knuth's TEX program and the LATEX set of macros, among others.

2. **Communications (via publication)**: Although no longer completely separable from the previous category, changes in how mathematical knowledge is formally communicated are manifold. The increased electronic distribution of papers is pressuring existing journals to adapt; as preprint distribution becomes the primary means of scholarly communication the mechanisms of peer review are in danger of being upset and the accepted importance of published results is being transformed; conference proceedings are being moved to all-electronic format for timely access and cost-effectiveness; conventional information content is being forced to make

[1] http://www.cecm.sfu.ca/organics/announcement/

room for alternate forms which are largely digitally generated, maintained and accessed. Examples include images, videos, animations, active tools/resources, audio, etc.

The impact upon the conventional publication industries will be dramatic and there is widespread concern for the groups who depend on those traditional means. These include publishers, authors and readers, although it is the publishers who will be most pressed to establish a viable role for themselves. Of particular concern for mathematics is the future of professional societies who depend on the revenue generated by journals.

3. **Transportation**: Cheaper air travel, and the granting structure that allows mathematicians to take advantage of it, have facilitated face-to-face meetings of mathematicians with each other and with users of mathematics. This seems to be the most effective method of generating interest in various methods or problems. However, this luxury may be endangered in the face of economic changes[2].

4. **Communications (between professor and student)**: We are now seeing a steady growth of the use of video, overhead display of computer screens, in-class use of calculators or hand-held computers, e-mail, newsgroups on USENET, and the Web. These are welcome changes from the blackboard which has so long dominated the transmission of mathematics from professors to students. Furthermore, there are several embryonic virtual universities in existence right now, radically changing the confines of the traditional classroom setting. Of course, these high-tech tools are not always appropriate, but they are particularly effective for communication of computational mathematics courses.

   Given economic pressures, we can calmly predict a reduction, perhaps even a drastic reduction, in the number of graduate and senior undergraduate mathematics courses offered at different universities. The students will instead use video conferencing equipment to take "courses" from professors at other networked universities[3]. The facilities are in place, the demands for cost-cutting are in place, and faculty retirements are taking place.

5. **Scientific Computation**, comprising both symbolic computation[4] and numerical computation[5], is at last coming into its own, both for research and teaching.

[2] http://camel.math.ca/NSERC/
[3] http://www.athena.edu/
[4] http://www.maplesoft.com/
[5] http://www.mathworks.com/

3 The Jungle of Mathematical Knowledge

Mathematics as a branch of human knowledge is several millenia old, and though each individual mathematician's historical contribution to this branch of knowledge may be small, the resulting edifice is grand indeed. It long ago ceased to be possible for a single mathematician to even know the basic definitions of every subfield of mathematical knowledge. We have reached the point of decay in some areas. Richard Askey has observed that Gregory Chudnovsky knows things about hypergeometric functions that no one has understood since Riemann and that, with Chudnovsky's eventual passing, no one is likely to understand again. The body of mathematical knowledge, organized into specialties and sub-specialties and sub-sub-specialties, none of which even use the same notation for similar ideas, has become a vast jungle.

The obvious success of mathematics makes this problem worse in a sense. New mathematical papers are being published every day, in journals almost too numerous to count, and certainly too numerous to read. New mathematical textbooks written at an undergraduate level arrive on the scene nearly as quickly. Moreover, even if no unfamiliar jargon and notation were being used, no one could possibly look through all this material to find what they wanted.

This ever increasing morass of mathematical information poses a considerable challenge for today's young mathematicians. The emerging digital and network technologies may provide some of the means for answering that challenge.

4 Technology and Access to Knowledge

Technology isn't always the blessing that we might wish for. It often obfuscates issues by introducing additional complications. On the other hand, it does some tasks exceedingly well. In particular, it stores, organizes and provides access to information in various forms. It can make it possible to grapple with quantities of information which on paper (or in the head) are overwhelming. Still, the impact of adopting technological solutions to the growing crisis of information overdose has yet to be clearly understood.

4.1 Encoding Mathematical Language

Publishing a mathematical paper in a reputable journal is one way of making mathematical knowledge accessible. People whose libraries contain that particular journal (one among tens of thousands of journals) read that paper, interpret the results and notation in terms of the problem under consideration, and implement any algorithms described. This of course leaves the work (and danger) of correctly inferring the intention behind the content to the reader, often an onerous, error-prone and inefficient task.

Surely it is better if the best algorithms and most useful mathematical knowledge are already implemented in a standard, widely available program for immediate use. At best, the mathematician user could well use the program without knowing all the technical refinements that make the computation possible—though in our view a basic knowledge on the part of the user is indispensable, providing confidence that the algorithms are not being totally umis-used. At worst, it provides a common basis or language for communication and exchange, one which has been carefully crafted for precision, accuracy and consistency, much like the traditional logical formalism behind conventional mathematics.

This appears to be what is happening today. A large portion of mathematical knowledge is being encoded into computer algebra systems and into numerical packages for scientific computation. How many mathematicians would be able to write a program which could evaluate the Jacobian elliptic functions efficiently and accurately over the complex plane? But once it's done, then everyone can use it (assuming the right computer with the right software with the requisite license).

It is this encoding of mathematical knowledge that we find the most significant aspect of modern technology. The next most significant is the exchange or communication of that encoded knowledge, for example the act of asking on the Internet for help on a problem—by keyword search or simply a query in the right newsgroup—is also extremely important. There is a tendency to drown in student-type questions (it's considered unethical to ask for outside help on assignments, for example, but it has been known to happen on some newsgroups) but nonetheless an important amount of research communication happens on the Internet.

4.2 Electronic Distribution of Papers

Papers are increasingly being distributed electronically. Originally driven by e-mail preprint servers and now by "document vault"-like, Web-based storage systems (and by a few electronic journals), these "e-prints" change the traditional way of funding scientific publication. Formerly, page charges for which professors were given grant money used to pay for publications. Increasingly now, computer networking (still often paid by grant money) is replacing physical publication.

However this distribution is still fragmented and piecemeal. The preprints distributed in this fashion are mostly unrefereed, and the archival value is often dubious. The computer system itself that you have stored your magnum opus on may be gone in a year or less—what happens to your paper, then? Worse, how do you refer to the wonderful preprints that you just read? And how reliable were they, anyway, being unrefereed?

5 Refereeing and Reliability

One clear side-effect of scientific communications going electronic is that maintaining the level of quality of mathematical publication will be difficult. Peer review processes have traditionally kept the standards high for printed materials. With everyone a publisher, it isn't clear how to do the same for electronic materials.

5.1 Our Process of Paper Selection

Each of the papers in this volume has been selected because the authors are 'well-known', and their reputation was a significant factor in this selection process. But just because someone has done superb work in the past does not mean that they are doing so now. Indeed, the very best of us make mistakes (in our view, the only people who make no mistakes are those who do nothing at all, for fear of making mistakes). So selecting authors on the basis of reputation is not enough. Additionally, many of the underlying papers have or will have appeared elsewhere, in refereed journals. In fact, we solicited particular papers from these authors, partly to ease the burden for the authors, partly to allow us to select papers with a good potential for "activation", and partly because the refereeing issue will be less urgent for such works: only updates or changes ought to need scrutiny (though even fatal mistakes will slip past good referees on occasion).

5.2 Our Input

For this *Proceedings* , each paper has been read very thoroughly by a minimum of two grad students or post-docs or faculty members. These readers varied in ability and background but gamely challenged anything they didn't understand or believe to be true.

Each of these readers had been asked to actively participate in the "activation" of the paper, looking closely for places to expand on the exposition and in particular for useful places for *live mathematics* to go. In these settings, Maple codes (or in some cases Axiom, GAP, Pari, or Mathematica codes) have been placed. Where possible the provided code is closely related to the problem under examination; if not an actual algorithmization of the underlying expressions. This permits the reader to investigate the claims and observations of the author in situ and perhaps to down-load code for further analysis.

Then there is editorial responsibility. Rob Corless, Jonathan Borwein, Peter Borwein and Loki Jörgenson are prepared to accept some part of the blame if something inappropriate slips through. In this particular hyper-context, the task of editing has taken on new dimension and demanded a fresh approach to an ill defined process. The editors may be reached for comment (or criticism) at omp@cecm.sfu.ca.

Finally, we have put in place annotation and submission mechanisms which will allow readers to comment on a paper and even to submit articles to the collection; naturally, these comments are available for other readers to see. In some sense this provides an open forum refereeing process.

5.3 The Role of Refereeing

The spectrum of activities that is called "refereeing" or peer review will still be necessary, perhaps more so than ever before given the "fire-hose" quantities of raw information the Web can deliver. Electronic journals and documents will need some mark of quality to stand out from the general stream, and traditional peer review may well play a role here, though one can imagine other ways of doing it (perhaps paying people to referee paper as some paper journals already do). Critical reviews and thoughtful summaries of forays into the stream, generated by people with judgement and taste (and a high capacity for winnowing the chaff), are also extremely useful.

This means providing software tools to make the process easier. Part of our goal for this project was to see how far we could advance without broaching this issue directly. Instead of creating software tools or mechanisms for refereeing, we provided interested and capable readers who peer reviewed somewhat in the traditional manner, except that they were known to the authors and worked directly with them to improve the papers. It is clear that this was a satisfactory way of preparing such conference proceedings but is equally clearly not feasible for most forms of publication.

6 Making Technology Work in our Favour

While information technologies are changing at a rate far beyond the capacity of individual users to assimilate, it is evident that current levels of sophistication allow for the creation of a supportive technological environment. What is called for at this stage is exploratory development to provide us with some necessary experience. This is one of the key aspects of Organic Mathematics Project[6] which produced the *Proceedings* .

6.1 The Ideal Environment

A mathematician working in ideal conditions would be able to look at a fresh problem and easily to access any related material, find all the work on simpler but similar problems, and quickly carry out any sub-computations needed for the solution of the fresh problem. Such a person also would be able to consult freely not only with colleagues, but with experts not previously known to the mathematician.

[6] http://www.cecm.sfu.ca/projects/OMP/

Naturally, a successful mathematician also needs the ability to generate fresh, workable, and appropriate ideas for each new problem, and this spark is to us the essence of good mathematics. Such a mathematician would surely thrive within the conditions outlined above. That said, there are formidable difficulties to providing and maintaining such an ideal environment.

6.2 What We Hope to Accomplish

What we want to accomplish with the Organic Mathematics Project is a more thoughtful use of the technologies available, moving towards the ideal described above. We are especially interested in the benefits of *integration* or of unity. We want the information in the *Proceedings* of this workshop to form examples of "living documents", connected to their references, connected to each other, connected to algorithms for live mathematical work on the part of the reader. We want them to be, in a word, "organic".

7 An Outline of the *Proceedings*

Upon connecting to the *Proceedings* , readers are immediately presented with a choice of paths to follow; they can proceed directly to the invited papers[7], browse through the exemplary pages from within the *Proceedings* , be introduced to the speakers, examine the objectives and issues of the Organic Mathematics Project, or access the many resources both directly and indirectly related to the papers. This presentation is less daunting for the non-expert readers of individual papers as it provides them with a rich background of material, thus allowing them to navigate through the papers without possessing at the onset a complete understanding of the mathematics involved. It also provides more expert readers with a vehicle for research and exploration.

7.1 The Speakers, the Talks, and the Papers

We have attempted to personalize the *Proceedings* by presenting a short biography of every speaker[8]. There are individual pictures accompanying these biographies as well as a collection of pictures taken during the conference. Also, a small segment of each Speaker's talk has been highlighted and can be viewed as an on-line video clip.

All the Invited articles in the *Proceedings* have been "activated" or "enlivened". In addition, so have some of the Associated articles. The degree of activation has depended on the nature of the material, the interest and skills of the authors and activators, and the exigencies and (time) constraints of the process. This process involves identifying the mathematics within formatted text,

[7] http://www.cecm.sfu.ca/organics/papers/
[8] http://www.cecm.sfu.ca/organics/authors/

and feeding it to an interactive mathematical platform (like Maple or Mathematica). The results are then displayed, allowing the user to manipulate the parameters within the mathematics to explore the content of the text. In addition, many hypertext links were included such as historical biographies, details of proofs of cognate results, and scanned images.

7.2 Related Resources and Links

Related material can be accessed by following some of the Related Links[9] which have been compiled both by the authors of individual papers and by the editors of the *Proceedings* . The Related Resources[10] offer a wealth of references, softwares and programs that can be used both as research and problem-solving tools.

Some readers may be interested in the history[11] of a particular problem, or the philosophical issues surrounding it. Many may be concerned with the impact of technology on mathematical research and publication. We encourage the exploration of such topics and have provided links to various relevant resources available on the Web.

8 The State of the Art this Day

8.1 Technical Elements of the *Proceedings*

The *Proceedings* has been constructed of as many different elements as were immediately available. This includes the full spectrum of multi–media as well as some elements of local design: Text is available as

- ASCII (or plain) text
- TEXand LATEX(pre–LATEX2ϵ) formatted text
- DVI
- PostScript
- HTML
- scanned images in GIF format

Images are available in-line or separately as

- GIF
- JPEG

[9] http://www.cecm.sfu.ca/organics/links.html
[10] http://www.cecm.sfu.ca/organics/resources.html
[11] http://www.cecm.sfu.ca/organics/philhist/

- PostScript
- PicTEX
- X bitmap

Animations are provided either as series of ordered GIF images or as MPEG/QuickTime format movies. In addition, interfaces have been developed for

- Maple
- Mathematica
- Gap
- document retreival
- document submission
- document annotation
- text search (Glimpse)

Most of these elements have been chosen, modified or organized for optimal use in HTML documents on the World Wide Web. This in itself both limits and defines how the elements can be used. As will be illustrated, this has posed a considerable challenge to the project developers faced with meeting the expectations of creative authors – without unlimited resources.

8.2 Hardware Used to Create this *Proceedings*

All of the project development took place at the Centre for Experimental & Constructive Mathematics. This is primarily a UNiX shop which is based on Silicon Graphics Inc. workstations. The full complement of machines includes

- SGI R4000 100 MHz Challenge servers
- SGI R3000 33 MHz Indigo workstations
- SGI R4600 PC 133 MHz Indy workstations
- SUN SPARC 20/61 server
- NCD X-terminals
- Mac 7100 A/V PC
- IBM 33 MHz '486 PC

Peripherals include

- Ricoh FS 2 colour scanner
- LaserWriter Pro 600 printer
- $\approx$ 10 Gbytes of available disk space

Typical memory on the workstations was around 24 – 35 Mbytes with a maximum of 192 Mbytes on one of the servers. At the time of this writing, the *Proceedings* consumed approximately $\approx$ 150 Mbytes.

8.3 Software Used to Create this *Proceedings*

While hardware is expected to remain useful on timescales exceeding ten years (PDP 11s and IBM 470s are still functional machines today), software evolves on timescales less than two or three years. Long before the machines stop running, companies stop providing support for their operating systems, development tools and environments, networking upgrades and other necessary software. In the case of networking tools related to the Internet (like World Wide Web), change is currently taking place on scales nearer to months. In addition, the size and complexity of the programming environments continues to increase as developers build for larger and larger memory, discounting the continued use of small memory machines.

8.3.1 Standard Tools, Software and Systems

All of the workstations that were used for the development run some variant of System V UNiX. The majority of those systems are IRIX v5.3 (SGI). The user environment is a C-shell variant within an X11 R5 windowing system. The system currently supporting the *Proceedings* is also IRIX v5.3.

Most aspects of the development environment are described in the following section. The server which provides access to the contents is a standard NCSA httpd daemon, v1.5, which has been configured to permit

- server push (Netscape extension)
- cgi-bin script execution
- server-side includes

No modifications or enhancements of the standard server-client http system were required.

8.3.2 Development Languages and Environment

The primary development languages are Hyper Text Markup Language (HTML), Perl (a scripting language similar to C-shell), and C/C++. Each provides a specific level of functionality:

- HTML version 2.0

  HTML (or Hyper Text Markup Language) is prehaps the most volatile resource that is involved. A key feature of the World Wide Web, the details of HTML's functionality, and the clients which interpret it, determine the look, feel and limitations of Web resources. While HTML v2.0 is the current standard, having replaced v1.0 only months ago, and is largely responsible for the popularity of the Web (thanks to in-line images, audio and certain formatting features), its limits have been the source of constant frustration for many developers. This is so much so that Netscape Communications, makers of the most popular browser Netscape Navigator, almost immediately introduced "extensions" to HTML which have subsequently caused many problems; many Web designers use the extensions (such as tables, background colours, and server/client push/pull) resulting in pages that can only be properly viewed by Netscape.

  An instantiation of SGML (Standard General Markup Language), HTML is composed of plain text "marked up" with special instruction elements which suggest how the text should be organized including text formatting, in-line images, page elements such as lines, and hypertextual links between different pages potentially on different servers across the Internet. When HTML-encoded information is delivered upon request, the client is free to interpret the elements howsoever it has been configured to. This makes quality control very difficult as there is little guarantee that the person viewing the information is seeing it as it was intended.

  The latest version of HTML has now been released, v3.0, and it includes provision for a new programming element, Java. We comment on the anticipated role of Java in a later section. We have not made any use of either HTML v3.0 or Java.

- Perl version 4.0

  Perl is an interpreted language optimized for scanning arbitrary text input, extracting information from the stream, and printing reports based on that information. It's a good language for many information management tasks. The language is designed to be practical (easy to use, efficient, complete) rather than aesthetic (tiny, elegant,). It combines many of the best features of the standard UNiX working environment like C, sed, awk, csh and sh. Similar to sh and csh, it is used as a scripting language much as that used for C-shell but it can be used in a structured fashion like C.

  This is the default language for responsive command "cgi-bin" scripts which can be attached to HTML pages. It allows for user tailored requests for information and is the basis for the various interfaces found in the Organics project (i.e. document submission). Unfortunately, most of the existing Perl-based technology is relatively immature and these interfaces have to be written entirely from scratch. In time libraries of useful routines are expected to appear. Perl code development consumes most of the developer time, closely followed by manual generation of HTML code.

- C/C++
  Since interpreted scripts can be often somewhat inefficient or insecure, C or C++ is used for all exectuables. In particular, these are most helpful for compute-intensive or memory-intensive operations and also for programs composed of highly complex operations or those needing a high level of security. Only a small percentage of the development was devoted to C/C++ coding.

8.3.3 Computer Algebra Systems

This volume uses several computer algebra systems to "activate" the papers, but principally uses Maple. There are several reasons for this, but the main one is that the CECM[12] is a "Maple shop". Most of us have used Maple for many years, and the Maple expertise locally available is by any standard considerable, with Michael Monagan (one of the Maple designers), Greg Fee, Simon Plouffe, and Rob Corless present and helping. Simon Fraser University has had Maple in its mathematics courses for many years, so there was a considerable level of background Maple ability in the grad students, post-docs, and faculty associated with the project as well.

That said, the Organic Mathematics Project is not limited to Maple, using Pari, GAP, Mathematica, and AXIOM, as well. Simon Plouffe has the most expertise with Pari of the people at the CECM, but one of our invited authors (Henri Cohen) is one of the principal designers of Pari. Stan Wagon is known for his Mathematica expertise, and Stan Devitt is a prime mover at Waterloo Maple Inc.

The main use of computer algebra in these papers is to enliven the examples. Many of the papers discuss algorithms, and it is reasonable to present the reader with an already-implemented version of the algorithm, both so that the reader may test the claims of the paper and so that the algorithm/code can be used by the reader in their own work.

Dave Fayegh had written a system that allows people to use Maple on our server from a remote site while reading one or more of these papers. This Maple Form Interface, though in its infancy, has proved already to be a valuable tool for mathematical exposition. If you try out some of the examples, we are sure you will agree. We note that there are similar form interfaces for other computer algebra systems, notably Mathematica, and that this is an idea whose time has come. We believe that our Maple Form Interface[13] is currently the best available, and we also believe that it is the most highly-developed from the point of view of serious use. The examples and algorithms displayed in the

[12] http://www.cecm.sfu.ca/
[13] http://www.cecm.sfu.ca/projects/OMP/nmpform.html

Maple annotation system (which uses the form interface) have been carefully chosen to make a difference in the understanding of the reader, not just in an attempt to impress the onlookers. (It is ironic that many of the least flashy parts of the /proceedings were the most difficult to provide.)

8.3.4 Document Delivery Systems

The Document Vault was intended to provide the basis for the flexible creation and use of on-line, multi-media documents. In particular, a mechanism for the inclusion of arbitrary types of information into a document is needed to support standard archiving and distribution practices on the Internet. The basic Vault features include:

- A definition of Vault objects composed of standard file types (images, sounds, text, LaTeX, data, multi-part, etc.) and appropriate headers.
- A definition of documents as one or more objects assembled into a particular structure.
- A definition of methods appropriate to each object necessary to manifest a document into
- Whatever target context is required (PostScript, HTML, ASCII, DVI, etc.).

Consequently, a document can be authored by including text, images, sounds, data, network pointers to the same or other forms of information and then served in any form required, for either local or network consumption. There is only one authorized document and so maintenance is limited to a minimum.

Originally, the Vault project was initiated to alleviate problems related to the maintenance and development of electronic archives. Anonymous FTP, Gopher and World Wide Web (WWW) archives are distinct in how the contents of each type of archive are typically organized. This is largely due to the nature of the underlying protocols. Moreover, the type of information varies: raw or compressed data files are the norm for FTP, ASCII text is the most useful for Gopher and HTML-structured multi-media is appropriate for the Web. Worse, the concept of a document is not clearly defined since a mix of images, sounds, text (ASCII and formatted) and data may be required. For a site offering services in each, problems of document authority, duplication and redundant maintenance chores are difficult to cope with.

The form in which a document appears in the *Proceedings* has been determined by archival demands, providing automated access to the various formats of the available documents through a single adaptable interface. Its potential as an aid to authoring was not explored as our focus was on the activation of the mathematical content.

8.3.5 Annotation System

The natural instinct for a person reading printed text is to write in the margin. This leaves a permanent record of the reader's impressions. Marginalia, however, only appears in the reader's copy of the text; no one else has access to that information. An annotation system connected to a public document, on the other hand, facilitates the process of both intellectual commentary and public dialogue. This is one of the apparent benefits of networking information.

The difficulty is in designing a system that lends itself to intuitive use. As suggested, the reader would normally scrawl something in a hardcopy margin. On screen, it must be typed. Further it has to be placed within the bounds of whatever representational system is in use. And it must be done in a way that doesn't interfere notably with original text. Current Web technology (HTML v2.0, cgi-bin scripts in Perl and browsers like Netscape v1.12) doesn't properly provide for the level of interactivity which would support an effortless notation system. Rather, more cumbersome work-arounds are necessary, in anticipation of Java and other environments which support true interactivity.

The system employed for the *Proceedings* is intended to work within the existing framework and provide a limited functionality. First, documents are marked up, either automatically or by hand, with icons indicating place markers where comments can be added. These are usually at the top of a page and around text which is anticipated to draw attention. Upon selecting the icon, the text page is entirely replaced with an interface which accepts the name of the reader and their comments. Upon completion, the comment text is stored alongside the document and an icon indicating the presence of the comment is added to the original text. Subsequent readers masy then choose to read the comment by selecting the new icon and, if desired, to supplement the comment with their own.

The shortcomings with this system are largely related to the break in flow which accompanies the addition of a comment. Since the Web currently functions under a form of batching mode, generating more or less static pages on demand, the reader cannot directly interact with the text or its environment.

8.4 Requirements for Access to the *Proceedings*

Optimally, the interested reader will have:

- direct access to the Internet (i.e. 10Mb/sec Ethernet)
- Netscape 1.1 or higher as a Web browser
- a current, fully equiped workstation or PC with 8-bit gaphics (or better)
- agents capable of interpreting most forms of multi-media to be found on the Web:

  - text formats - DVI, PostScript, etc.
  - audio - WAV, AU, etc.
  - movie - MPEG, QUICKTIME, etc.
  - images - GIF, JPG, TIFF, RGB, XBM, etc.

Minimally, a reader must have:

- low speed access to the Internet (eg. 14.4K modem)
- networking support (i.e. SLIP, PPP)
- a text-based Web browser

In the minimal case, the content is still largely accessible except for the HTML-based representations which rely on in-line graphics. In many cases, the benefits of the enhancements are also lost since they employ graphics as well. The unenhanced documents are, however, downloadable.

8.5 The Technological Lifetime of the *Proceedings*

The underlying technologies are rapidly being replaced by the next generation's developments. These include the change from HTML v2.0 to v3.0 (which supports mathematical fonts), the Java environment (which supports real interactivity), the addition of new extensions for browsers like Netscape, the augmentation and adaptation of software like Maple for direct use on the Internet and other significant evolutionary advances. The prospect for the long term of the *Proceedings* is certainly obsolesence if no efforts are made to upgrade it appropriately.

But this is true of almost any piece of software or digital resource. It happens to be particularly so in the case of Web-related materials at this point in the development of the Internet. Without constant upkeep, the state-of-the-art is expected to fall to the status of unexceptional by the summer of 1996 and to that of obsolence by late 1997. However, like FTP and gopher resources, it is anticipated to continue to be useful and accessible (if not a bit "antiquated") until after the year 2000.

9 The Future of these *Proceedings*

We have some concerns about the archival value of this volume. Paper is a durable medium, in its acid-free state. How durable is software? These *Proceedings* are composed of HTML code bound together with CGI bin scripts connected to current versions of mathematical software (like Maple or Mathematica), together with a collection of images, data and text in various formats from GIF to Postscript to ASCII to raw binary. Where will this volume be in ten years? How about ten months? The standards are changing very rapidly. Experience to date tells us that the potential for long-term functionality is quite limited. Constant improvements cause old versions of any software, whether programming language or text formatting, to be discarded and forgotten. The storage medium is also volatile as the supporting hardware technology shifts out from under it.

Perhaps a more pertinent question is, supposing that someone will be able to read these *Proceedings* at some time in the future, will they want to? Although this experiment is at the limit of today's technologies, in some not so distant tomorrow it may be as "low fidelity" as old 58 rpm wax phonograph records compared to CDs. Certainly the relevance of the content will endure but the delivery mechanism may impact on its effectiveness.

In a somewhat retrograde act, the Canadian Mathematical Society[14] has published the *Proceedings* in the traditional hardcopy form. This seems appropriate insofar as the hardcopy may act as a telltale in the distant future, much as the skeletal remains of an organism which has long since turned to dust. It will carry a description of the means and function of an experiment at a critical turn in the history of mathematical communications.

While this point of view offers a somewhat dim future for the *Proceedings* , it doesn't detract from the underlying value of both the content and the experience. Like many things at the bleeding edge of development, its basic importance is in illuminating the path to the immediate future.

10 The Most Valuable Lesson

Regardless of whether the *Proceedings* are functionally useful in a year or a century from now, the process of its creation and development has been a tremendously valuable experience. At a time in the history of communications when technology is having a real-time impact on how people express themselves, we have had an opportunity to examine in detail the practical issues behind digital publishing and integrated network environments.

The Organic Mathematics Project (OMP) was directed towards the exploration of the emerging network and information technologies within the context of mathematics. Numerous groups around the world are engaged in enhancing the specific aspects of the information highway and its associated processes for transporting data. However relatively few are actively integrating and adapting the raw technological building blocks to suit particular fields of endeavour such as Mathematics. In our case, we incorporated several different mechanisms into a single coherent environment which supports the contributions and interactions between mathematics researchers inclined towards experimental mathematics.

Many things which seemed self-evident theoretically became problematical in application. Issues related to copyright which had been carefully avoided

[14] http://camel.math.ca/CMS/

in advance came back to hold up publication. The limitations of the software necessitated taking some shortcuts (or even dropping whole concepts). The requests of the authors, who couldn't know clearly what was possible and what was not, often drove development in unexpected directions. Issues of compatibility severely limited the graphic potential of presentation. And of course time was limited.

What was left after all was done was a much clearer sense of what was possible. And what will be possible in the near future. Not suprisingly, the hype surrounding the Internet and the World Wide Web is intense. But there is little evidence to support the conjecture and the speculation about what it really is and what it is good/bad for. Especially with regard to very important issues surrounding publishing, education, scientific communication and virtual research environments. Our experience with these *Proceedings* has provided valuable feedback for subsequent steps toward an effective integrated on-line environment.

11 What Next?

The Organic Mathematics Project has taught us, above all, that it is possible and desirable to enliven mathematics exposition with appropriate technology. Some of the papers are very clearly much better for this process. We also learned something of the limitations of current technology. Two of our biggest gripes are with HTML and with the poor quality resolution of the typical computer screen.

As a permanent archival document this *Proceedings* does not really succeed, though it comes closer than we thought it would. As a prototype and an experiment to show what was possible and what is useful, and as a positioning tool to help us get ready for the real revolution to come, it succeeds far beyond our expectations.

We believe this project to be only the beginning of a revolution in mathematics education. It has barely *touched* the issue of multimedia (though we are making an attempt to include some videos of the speakers). The technology is changing rapidly, and we believe that very soon the issues of multimedia, living interactive mathematics using animation and real human contact, live on the Internet, will become inescapable.

Again, we do not believe that this *Proceedings* is the last word. We want to move from the research exposition end to the undergraduate education end (some concession to that is made in this volume, it is true), and perhaps even into the high schools. We want more living and lively mathematics—and we think we'll get it.

These are exciting times to live in.

12 Acknowledgements

This project is a joint effort. The principal technical contributors were David Fayegh, Jack Ho, Greg Fee and Brigitte Dorner. The authoring team members who worked with the contributing authors were Dhavide Aruliah, Heinz Bauschke, David Bradley, Luis Goddyn, John Hebron, Daryl Hepting, Veso Jungic, Michael Monagan, Andrei Oltean, Jennifer Overington, Chris Pinner, Simon Plouffe, Lesley Robinson, Xianfu Wang, Erick Wong, Philip Ye. The executive coordinator was Nathalie Sinclair.

For the editors of these *Proceedings* , it has been a real pleasure to work with this group of people. Especially considering the experimental nature of the project, the level of excellence demonstrated by each member of the team has been truly gratifying. The quality of the *Proceedings* clearly attests to their abilities.