

6. If $g(x) = \int_{-1}^x \sqrt{t^3 + 1} dt$ then $g'(x) = \sqrt{x^3 + 1}$.

10. If $F(x) = \int_x^4 (2 + \sqrt{u})^8 du = -\int_4^x (2 + \sqrt{u})^8 du$ then $F'(x) = -(2 + \sqrt{x})^8$.

16. If $y = \int_{-5}^{\sin x} t \cos(t^3) dt$ then $\frac{dy}{dx} = \sin x \cdot \cos(\sin^3 x) \cdot \cos x$
by the Chain Rule and Part 1 of the Fundamental Theorem of Calculus.

20. $\int_1^2 (5x^2 - 4x + 3) dx = \left(\frac{5}{3}x^3 - 2x^2 + 3x \right) \Big|_1^2 = \frac{26}{3}$.

40. $\int_{-5}^{-2} \frac{x^4 - 1}{x^2 + 1} dx = \int_{-5}^{-2} (x^2 - 1) dx = \left(\frac{1}{3}x^3 - x \right) \Big|_{-5}^{-2} = 36$.

46. $\int_{\pi/4}^{\pi} \sec^2 \theta = \tan \theta \Big|_{\pi/4}^{\pi} = -1$ is **incorrect**.

Integrating from left to right, how could a positive integrand yield a negative integral?
The Fundamental Theorem of Calculus does not apply, because of the infinite discontinuity of $\sec^2 \theta$ at $\theta = \frac{\pi}{2}$.

The integral $\int_{\pi/4}^{\pi} \sec^2 \theta$ does not exist.

48. $\int_{\ln 3}^{\ln 6} 8e^x dx = 8e^x \Big|_{\ln 3}^{\ln 6} = 24$.

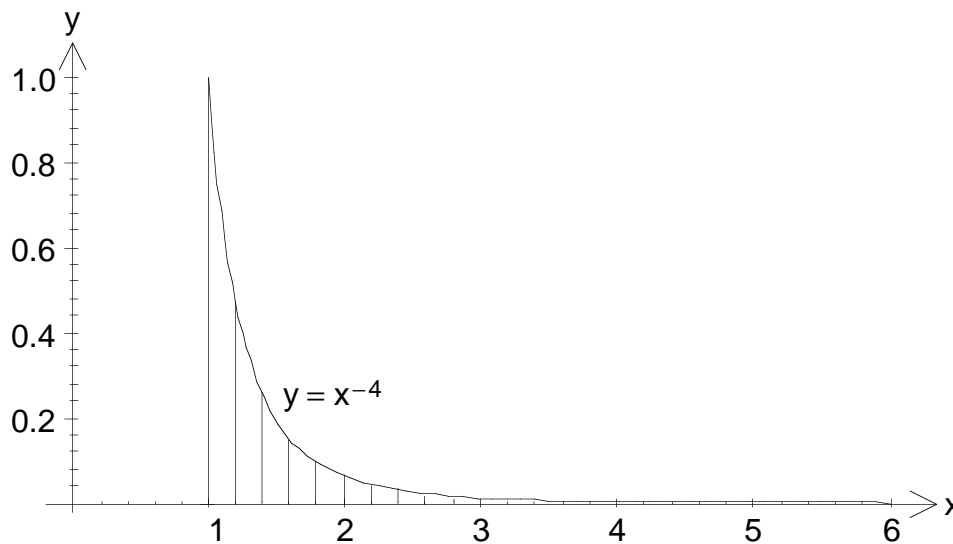
50. $\int_{-e^2}^{-e} \frac{3}{x} dx = 3 \ln|x| \Big|_{-e^2}^{-e} = -3$.

52. $\int_0^{0.5} \frac{dx}{\sqrt{1-x^2}} = \sin^{-1} x \Big|_0^{0.5} = \frac{\pi}{6}$.

60. If $f(x) = \begin{cases} x & \text{if } -\pi \leq x \leq 0 \\ \sin x & \text{if } 0 < x \leq \pi \end{cases}$

then $\int_{-\pi}^{\pi} f(x) dx = \int_{-\pi}^0 x dx + \int_0^{\pi} \sin x dx = \left[\frac{1}{2}x^2 - \cos x \right]_{-\pi}^{\pi} = -\frac{1}{2}\pi^2 + 2$.

62. To the right is a graph of $y = x^{-4}$, $1 \leq x \leq 6$, with subdivisions at intervals of 0.2 along the x-axis and at intervals of 0.04 along the y-axis. Vertical line segments are drawn for $1 \leq x \leq 3.4$; after that the segments are too short to see.



Using my TI-36 calculator I found the sum of the areas of the trapezoids approximating the indicated vertical strips; the first trapezoid top for instance goes from $(1, 1)$ to $(1.2, 1.2^{-4})$. The result I obtained is 0.338651206 which I will round off to 0.34. Notice that the tops of the trapezoids are outside the region since the curve is concave upwards, and this means I am overestimating the area inside each strip. I am neglecting the strips to the right of $x = 3.4$ (which don't contribute much area).

Area = $\int_1^6 x^{-4} dx = -\frac{1}{3} x^{-3} \Big|_1^6 = \frac{215}{648} \approx 0.331790123$. The approximation is rather good!

74. $\int (\cos x - 2 \sin x) dx = \sin x + 2 \cos x + C$.

78. (a) If $v(t) = (t^2 - 2t - 8)$ m/s, $1 \leq t \leq 6$ gives the velocity then the displacement is $s(6) - s(1) = \int_1^6 (t^2 - 2t - 8) dt = \left(\frac{1}{3} t^3 - t^2 - 8t \right) \Big|_1^6 = -\frac{10}{3}$ m.

(b) $v(t) = (t - 4)(t + 2)$ so $v(t) \leq 0$ for $1 \leq t \leq 4$ and $v(t) \geq 0$ for $4 \leq t \leq 6$. The distance traveled is $\int_1^6 |t^2 - 2t - 8| dt = \int_1^4 (-t^2 + 2t + 8) dt + \int_4^6 (t^2 - 2t - 8) dt = \left(-\frac{1}{3} t^3 + t^2 + 8t \right) \Big|_1^4 + \left(\frac{1}{3} t^3 - t^2 - 8t \right) \Big|_4^6 = 18 + \frac{44}{3} = \frac{98}{3}$ m.

80. (a) If $a(t) = (2t + 3)$ m/s² and $v(0) = -4$ m/s, $0 \leq t \leq 3$, $v(t) = (t^2 + 3t - 4)$ m/s.

(b) $v(t) = (t + 4)(t - 1)$ so $v(t) \leq 0$ for $0 \leq t \leq 1$ and $v(t) \geq 0$ for $1 \leq t \leq 3$. The distance traveled is $\int_0^3 |t^2 + 3t - 4| dt = \int_0^1 (-t^2 - 3t + 4) dt + \int_1^3 (t^2 + 3t - 4) dt = \left(-\frac{1}{3} t^3 - \frac{3}{2} t^2 + 4t \right) \Big|_0^1 + \left(\frac{1}{3} t^3 + \frac{3}{2} t^2 - 4t \right) \Big|_1^3 = \frac{13}{6} + \frac{38}{3} = \frac{89}{6}$ m.

82. The increase in population is $\int_4^{10} (200 + 50t) dt = (200t + 25t^2) \Big|_4^{10} = 3300$.

86. If $g(x) = \int_{\tan x}^{x^2} \frac{1}{\sqrt{2+t^4}} dt$, then $g'(x) = \frac{1}{\sqrt{2+x^8}} \cdot 2x - \frac{1}{\sqrt{2+\tan^4 x}} \cdot \sec^2 x$.

To see why, write $g(x) = \int_a^{x^2} \frac{1}{\sqrt{2+t^4}} dt - \int_a^{\tan x} \frac{1}{\sqrt{2+t^4}} dt$ for some constant a , and apply the Chain Rule and Part 1 of the Fundamental Theorem of Calculus.

44. $\int_3^4 f(x) dx + \int_1^3 f(x) dx + \int_4^1 f(x) dx = \int_1^3 f(x) dx + \int_3^4 f(x) dx + \int_4^1 f(x) dx =$
 $= \int_1^4 f(x) dx + \int_4^1 f(x) dx = \int_1^4 f(x) dx - \int_1^4 f(x) dx = 0$.

48. $\int_{-3}^5 f(x) dx - \int_{-3}^0 f(x) dx + \int_5^6 f(x) dx = \int_{-3}^5 f(x) dx + \int_0^{-3} f(x) dx + \int_5^6 f(x) dx =$
 $= \int_0^{-3} f(x) dx + \int_{-3}^5 f(x) dx + \int_5^6 f(x) dx = \int_0^5 f(x) dx + \int_5^6 f(x) dx = \int_0^6 f(x) dx$.

50. $\sqrt{5-x} \geq \sqrt{5-2} = \sqrt{3} = \sqrt{2+1} \geq \sqrt{x+1}$ if $1 \leq x \leq 2$.

Therefore $\int_1^2 \sqrt{5-x} dx \geq \int_1^2 \sqrt{x+1} dx$ by Property 7 on Page 343 of the text.

56. If $0 \leq x \leq 2$, then $1 \leq \sqrt{x^3+1} \leq 3$, so $1 \cdot (2-0) \leq \int_0^2 \sqrt{x^3+1} dx \leq 3 \cdot (2-0)$
 by Property 8 on Page 343, so $2 \leq \int_0^2 \sqrt{x^3+1} dx \leq 6$.

The approximate value of the integral $\int_0^2 \sqrt{x^3+1} dx$ is 3.2413092631952725570, according to *Maple* on my Macintosh.

I do not know how to evaluate this integral exactly.

62. If $2 \leq x \leq 5$, then $\sqrt{x^2-1} \leq x$, so $\int_2^5 \sqrt{x^2-1} dx \leq \int_2^5 x dx = \frac{1}{2} (5^2 - 2^2) = 10.5$.

In fact the exact value of the integral $\int_2^5 \sqrt{x^2-1} dx$ is $5\sqrt{6} - \sqrt{3} + \frac{1}{2} \ln \frac{2+\sqrt{3}}{5+2\sqrt{6}}$, which is approximately 10.027661020028832708.